Problem #1 is from section 1.2, #10. Which was on our first homework.

#2 This problem is incorrect as \( K = \emptyset \).
To see this let \( x \in \emptyset \) and we will show that \( x \in K \).

Since \( L = \emptyset \) is a Dedekind cut, \( \emptyset \neq 0 \)
so \( 3 x \in L \). If \( x + y < 0 \) then \( x \in K \)
and we are done. If \( x + y \geq 0 \)
then \( x + y + 1 > 0 \) so \( y - (x + y + 1) < y \)
and \( \{ y - (x + y + 1) \} \). \( L \).
Since \( x + (-x - 1) = -1 < 0 \) this implies \( x \in \emptyset \).

So \( x \in K \).

#3 Let \( r = \frac{p}{2} \) where \( p \) & \( q \) are relatively prime. Then

\[
2 \left( \frac{p}{2} \right)^n + a_{n-1} \left( \frac{p}{2} \right)^{n-1} + \ldots + a_0 = 0
\]

so \( 2 \left( \frac{p}{2} \right)^n = \left( a_{n-1} \left( \frac{p}{2} \right)^{n-1} + \ldots + a_0 \right) \).
If we multiply both sides by $q^n$ we have

$$2p^n q^\frac{n}{2} = -q^n (a_{n-1} \left(\frac{r}{q}\right)^{n-1} + \ldots + a_0).$$

Note that

$$K = -q^{n-1} (a_{n-1} \left(\frac{r}{q}\right)^{n-1} + \ldots + a_1)$$

is an integer so

$$2p^n = qK.$$

Assume $n > 1$.

Let $q'$ be a prime such that $q' | q$.

Then $q' | 2p^n$ or $q' | r^n$.

If $q' \mid r^n$ then $q' | p$ but since $q$ and $p$ are relatively prime $q' \nmid p$.

We therefore let $q' = 2$.

Divide both sides of $2p^n = qK$ by $2$.

we see $n \leq q = \frac{q_2}{2}$. Where $q_2$ is an integer. If $q_2 \neq 1$ then we exact another prime $q''$ s.t. $q'' \mid q_2$. But $q'' \mid 2$ (since $q'' = 2$) and $q'' \mid p^n$ or $q'' \nmid p$. Again since $p$ and $q$ are relatively prime this is impossible so we must have $q_2 = 1$. 
We have shown that $q = 1$ or $q = 2$. Therefore $2^\frac{p}{q}$ is an integer.