QUIZ 5 – March 28, 2001
Take Home Quiz

1. Find all critical points, inflection points and zeros of
\( f(x) = x^3 - 3x^2 + 3x + 10 \) and use this information to sketch its graph.
Also, where are the local minima and maxima on \([-2, 4]\).

To find zeros, set \( f(x) = 0 \) and solve, Notice, \( x^3 - 3x^2 + 3x = (x - 1)^3 + 1 \),
so we have \( (x - 1)^3 = -11 \). Solving this we get \( x = \sqrt[3]{-11} + 1 \).

To find critical points, we need to solve \( f'(x) = 0 \). So, we take the derivative of \( f(x) \) to get \( f'(x) = 3(x^2 - 2x + 1) = 3(x - 1)^2 \). Setting this equal to zero and solving, we get \( x = 1 \) as a critical point.

To find possible inflection points, we need to solve \( f''(x) = 0 \). Taking the derivative of \( f'(x) \) we get that \( f''(x) = 6x - 2 \). Setting this equal to zero and solving yields \( x = 1/3 \).

The graph looks like

2. For the function \( g(x) = |x| \) on the interval \([-2, 2]\), the Mean Value Theorem
would say that there exists a point \( c \in (-2, 2) \) where \( g'(c) = 0 \). Is this true for the given function \( g(x) \)? And if not, why does this not contradict the Mean Value Theorem?
No, this is not true for \( g(x) \), and this does not contradict the Mean Value Theorem because \( g(x) \) is not continuous.

3. Find \( f(x) \) given that \( \frac{df}{dx} = 3x + \sqrt{x} \). What happens to the arbitrary constant from the first integration?

Integrate once to get
\[
\frac{df}{dx} = \frac{1}{2}x^2 + \frac{2}{3}x^{3/2} + C_1
\]
Integrate a second time to get
\[
f(x) = \frac{1}{3}x^3 + \frac{4}{15}x^{5/2} + C_1x + C_2.
\]
The constant from the first integration is integrated to get \( C_1x \).

4. Prove the formula \( \int [f'(x)g(x) + g'(x)f(x)] \, dx = f(x)g(x) + C \). Use the definition of antiderivative.

The definition of antiderivative is that \( F(x) \) is an antiderivative of \( f(x) \) if \( F'(x) = f(x) \). Therefore, \( f(x)g(x) \) is an antiderivative of \( f'(x)g(x) + g'(x)f(x) \) because \([f(x)g(x)]' = f'(x)g(x) + g'(x)f(x)\) because \([f(x)g(x)]' = f'(x)g(x) + g'(x)f(x)\)