Quiz 2 — 15 Points  
Feb. 2, 2001

1. (5 points) Give a written definition for \( \lim_{x \to c} f(x) \). I’m not looking for a definition in terms of \( \epsilon \) and \( \delta \), but what it means in words.

\( \lim_{x \to c} f(x) \) represents the number (if there is one) that \( f(x) \) comes arbitrarily close to as \( x \) gets close to \( c \).

2. (5 points) Show the following using an \( \epsilon - \delta \) proof

\[
\lim_{x \to 3} (-2x + 7) = 1
\]

We start with \(|f(x) - L| < \epsilon\) and manipulate it to get \(|x - c| < \delta\)

\[
|f(x) - L| = | -2x + 7 - 1 | = | - 2(x - 3) | = | - 2||x - 3| = 2|x - 3| < \epsilon
\]

Therefore, for any \( \epsilon \) we can take \( \delta < \epsilon/2 \) to guarantee that \((-2x + 7) - 1| < \epsilon \) if \(|x - 3| < \delta\) which is the definition of the limit.

3. (5 points) Find the following limit using any rule we’ve shown in class. It might help to use some trigonometric identities first.

\[
\lim_{x \to 0} \left( \frac{2x \cot(x)}{\cos(x)} \right)
\]

Recall, \( \cot(x) = \cos(x)/\sin(x) \) so

\[
\lim_{x \to 0} \left( \frac{2x \cot(x)}{\cos(x)} \right) = \lim_{x \to 0} \left( \frac{2x \cos(x)}{\sin(x) \cos(x)} \right) = \lim_{x \to 0} \left( \frac{2x}{\sin(x)} \right) = 2 \lim_{x \to 0} \left( \frac{x}{\sin(x)} \right) = 2
\]