Tips for Succeeding in Foundations of Analysis

Mathematics as a Language

Mathematics is a language just like English, German, Japanese, or Esperanto. Like these languages, mathematics has a vocabulary, syntax, and grammar. When reading and writing mathematical papers, we will do so in one of these underlying languages (English, German, etc). Therefore, some of the vocabulary, syntax, and grammar will come from whatever the underlying language. However, mathematics also has some universal vocabulary, syntax, and grammar. For example, when one says a function is continuous, we understand that to mean continuous in the mathematical sense, rather than take the definition from whatever language it is we are writing in. This course will teach us some of this universal mathematical vocabulary, and how to use it in mathematical sentences (theorems).

Vocabulary: Definitions

If we were learning French, we would first need to build a vocabulary. We won’t be able to write any sentences down without first having some words and their definitions at our disposal. Oh, we could carry a dictionary around with us, but that does not make for successful conversations. It is better to simply know that “chien” means “dog”. Likewise in mathematics. Therefore, I recommend that you develop a dictionary of mathematical definitions, all compiled together and arranged by chapter. You should refer to this dictionary when absolutely necessary, but you should also try to commit these definitions to memory. You may feel that you understand the definitions and that they “make sense”, but you really need to “know” the definitions and their precise meanings. Test yourself on these, use flash cards, or simply practice writing them down.

Sentences: Theorems

In this course, we will be reading and writing mathematical theorems and their proofs. If definitions are the vocabulary of the mathematics language, then theorems are the sentences. In fact, the German word for theorem is “satz”, which is the same as the German word for sentence. Like the definitions, you should commit the theorems to memory. It is not necessary to commit to memory their proofs, but you should try to remember the single idea behind the proof. In this way, you can often recreate the proof on demand. Don’t worry, you won’t be asked to recreate the proof for any of the big theorems (yet! If you are a math major, you will probably have to at some point). Again, drill yourself on the theorems, as they will be useful in your homework problems.