Sets and Functions

Recall the “Image/Pre-Image Theorem” from class:

**Theorem (Image/Pre-Image).** Suppose that \( f : A \rightarrow B \). Let \( C, C_1, \) and \( C_2 \) be subsets of \( A \), and let \( D, D_1, \) and \( D_2 \) be subsets of \( B \). Then the following are true:

1. Complete the proofs of the following parts of the Image/Pre-Image Theorem.

   **Proof.** (c) Let \( y \in f(C_1 \cap C_2) \). Then there exists a point \( x \) in \( C_1 \cap C_2 \) such that \( f(x) = y \). Since \( x \in C_1 \cap C_2 \), \( x \in \) \( \) and \( x \in \) \( \). But then \( f(x) \in \) \( \) and \( f(x) \in \) \( \), so \( y = f(x) \in \) \( \).

   (f) Let \( x \in f^{-1}(D_1 \cup D_2) \). Then \( f(x) \in \) \( \), so \( f(x) \in D_1 \) or \( f(x) \in D_2 \). If \( f(x) \in D_1 \), then \( x \in \) \( \). If \( f(x) \in D_2 \) then \( \). In either case, \( x \in f^{-1}(D_1) \cup f^{-1}(D_2) \).

   Conversely, suppose \( x \in \) \( \). Then \( x \in f^{-1}(D_1) \) or \( x \in f^{-1}(D_2) \). If \( x \in f^{-1}(D_1) \), then \( f(x) \in \) \( \). If \( x \in f^{-1}(D_2) \), then \( f(x) \in \) \( \). In either case, \( f(x) \in D_1 \cup D_2 \), so that \( x \in \) \( \).

2. Draw pictures to help you understand each part of the Image/Pre-Image Theorem.