HW #2 – MATH 6320
FALL 2024

DUE: MONDAY, JANUARY 31

1. (a) If $H \subset G$ is a subgroup and $f : G' \to G$ is a group homomorphism, show that $f^{-1}(H) \subset G'$ is a subgroup.

(b) If $N \subset G$ is a normal subgroup, show that $f^{-1}(N) \subset G'$ is normal.

2. Prove the Second Isomorphism Theorem for Groups:
If $H \subset G$ is a subgroup and $N \subset G$ is a normal subgroup, then:

$$\frac{H}{H \cap N} \text{ and } \frac{HN}{N} \text{ are isomorphic groups}$$

(see the first problem set for much of what you need for this theorem)

3. If $H \subset G$ is a subgroup and $|G/H| = m$, show that:

(a) left multiplication:

$$l : G \to \text{ Perm}(G/H) \cong S_m;\ l(g) = l_g$$

is a transitive action and $H \subset G$ is the stabilizer of the coset $H \in G/H$.

(b) If $m \geq 3$, show that $l(G) \cap A_m \neq \{e\}$.

(c) If G is simple (i.e. G has no normal subgroups other than e and G), show that l is injective and $l(G) \subset A_m$, so

$$|G| \text{ divides } |A_m| = m!/2, \text{ the order of the alternating group}$$

4. Find all the subgroups of A_4 and specify which of them are normal.

5. Find an explicit surjective homomorphism:

$$f : S_4 \to S_3$$

i.e. tell me where f takes (1 2), where it takes (1 2 3), etc.

6.[1] Convince yourself that:

(i) A_4 is the group of rotational symmetries of the regular tetrahedron

(ii) S_4 is the group of rotational symmetries of the cube, and

(iii) A_5 is the group of rotational symmetries of the dodecahedron

[1]not to be turned in