Math 6180/Topics in Complex Geometry/Spring 2020
Syllabus

Class meets: MW 3:00-4:20 in LCB 215

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This is a course on closed Riemann surfaces and finite metric graphs.

A Riemann surface has the topology of a sphere or a "g-holed torus." In the former case the topology of the sphere determines the surface. In the latter case, there are *moduli* to the locus of non-isomorphic Riemann surfaces with the topology of a given g-holed torus. We will investigate these Riemann surfaces via their meromorphic functions and differentials and their associated "linear series" that map the Riemann surfaces to projective space, converting it from an "analytic" object to an "algebraic" object, defined by the zeroes of polynomial equations.

In parallel, we'll study finite graphs. Graphs also have a genus g, that can be seen by collapsing spanning trees to a bouquet of circles. There is a surprising, parallel to the study of meromorphic functions on a Riemann surface given by a "chip-firing" game on a graph. From this point of view, linear series translate to equivalence classes of chip configurations, and many results about Riemann surfaces have analogues in the world of graphs. To convert the graphs to algebraic objects, we will use the semi-ring of **tropical** numbers.

References for Riemann surfaces:

[ABCH] Arbarello et al, Geometry of Algebraic Curves, Volume I
[Mi] Miranda, Algebraic Curves and Riemann Surfaces,
[Mu] Mumford, Algebraic Varieties I, Complex Projective Varieties

References for Graphs:

[BN] Baker/Norine, Riemann-Roch and Abel-Jacobi on a finite graph.
[BN2] Baker/Norine, Harmonic morphisms and hyperelliptic graphs.