Factoring Quadratic Polynomial Notes

Quadratic - an equation involving the second (and no higher) power of an unknown quantity.

Polynomial - consisting of more than two algebraic terms.

Monomial - one algebraic term.

Quadratic Polynomial - mathematical expression involving the second (and no higher) power of an unknown quantity, and at least one other term. It is an expression of the form ax2+bx+c = 0, where a 0.

Quadratic Polynomial (2nd Definition)- a mathematical expression using exponents on variables with a degree to the second power, and no higher power. We will discuss quadratic polynomials with only one variable.

Factor - sequences of numbers or expressions that can be multiplied together to get another number or expression.

a. Examples
    i. 4 is a factor of 8
    ii. x + 3 is a factor 3x + 3
    iii. x – 1 is a factor of x2– x – 2
    iv. 8

        ^

        4 2

        ^

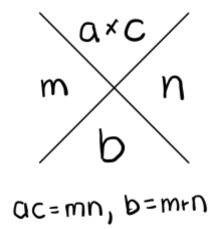
        2 2

Factoring (2nd Definition) - breaking down an expression into terms that can be multiplied back together to produce the original expression.

Ways to Find Where an Equation Equals Zero

1. Common Factor - Common factor problems will either be an integer, or an integer times a binomial or polynomial. This will allow you to find rational roots.
    a. Examples
        i. (6x2-2x) = 2x(3x-1)
        ii. 128=2(64)=2(8)(8)
2. Using a Graph - Graphing an equation can show you where you have zeroes, including where an equation has irrational zeros. Finding irrational zeros cannot be done by factoring, meaning that sometimes graphing will be necessary.

3. Using the Quadratic Formula - The quadratic formula is a method to find the zero roots of any line in the form ax2+bx+c = 0, where a0.
4. Factor by Grouping - This is also referred to as the cross method. In order to do this, we take an equation in the form of ax2+bx+c=0, and split it into ax2 +mx+nx+c=0, where m+n=b. Now, how do we go about determining our m and n values? That is where the cross part comes in. For this, we will look at the value of ac and b. We want to find two numbers that multiply to the value of ac and add to the value of b. These two numbers will be our m and our n values. An important note is m and n may be positive or negative values. The reason why it is called the cross method is because it is typically drawn as follows:



This method is often used with the box method for factoring quadratic polynomials. With the box method, we will put the four pieces of our equation, ax2, mx, nx, and c into a box as shown below:

ax^2	mx
nx	c

Now, we take each column, and look at what the two values of those columns have in common. If we had a column with 2x2 and 6x, the common factors of each value are 2x. Now, we take that common value, and write it above the columns. Then we look at the rows and find the common factors in the two values. Then we write these values on the right. In the end, we will get something like the example below:

	$2x$	-1
x	$2x^2$	$-3x$
3	$6x$	-3

For those wanting a proof of the box method, Professor Bertram mentioned he could post it on Canvas.

History

Around 600 BC, Indian mathematician Brahmagupta wrote down a formula to solve any quadratic equations. His most general solution was very similar to the quadratic formula. This solution is as follows, “To the absolute number multiplied by four times the [coefficient of the] square, add the square of the [coefficient of the] middle term; the square root of the same, less the [coefficient of the] middle term, being divided by twice the [coefficient of the] square is the value.” This is equivalent to the formula we have today to solve the equation.

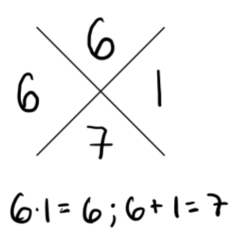
He came up with four different ways that the quadratic equation can be written:

5. ax2+c =bx
6. ax2= bx +c
7. ax2+bx +c =0
8. ax2+bx=c

Example Problems

9. 2x4+7x2+3

Now, this problem can seem complicated because students may not be used to working with x4, but this problem is really just a quadratic polynomial in disguise. We can use the cross method and the box method to factor by grouping.

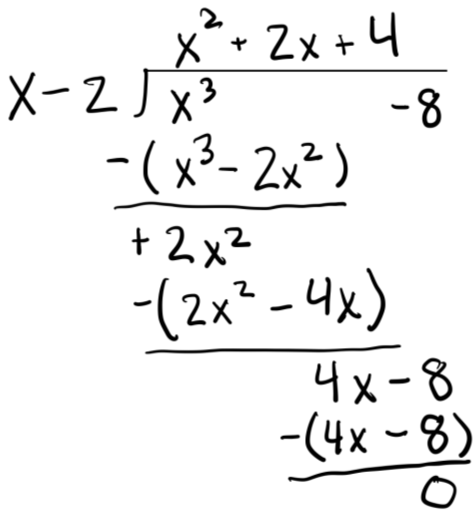


So now we see that 2x4+7x2+3=(2x+1)(x+3).[5]

	2x	1
x	$2x^2$	1x
3	6x	3

Difference of Two Cubes

When trying to factor a set of cubes in the form, a3-b3, we are guaranteed that a-b is a factor. In order to see what other factors a3-b3 has, we will need to perform long division. Let us try doing this with the equation x3-23:



So now we see that x3-23=(x-2)(x2+2x+4). So x=2 is a root of x3-23, and x-2 is a factor of x3-23.

Higher Degree Perfect Square

10. x4-1

This problem again can seem challenging as it is in a higher degree, but analyzing this expression closely we can see that the polynomial is a perfect square so it simplifies down to (x2-1)(x2+1).

So now (x2-1) is also a perfect square which simplifies into (x-1)(x+1)

As (x2+1) is non factorable in the real plane, because of the graph not hitting the x-axis, the factors are as follows: (x2+1)(x-1)(x+1).