Quiz Scores (out of 10): n = 133; Mean = 5.7; median = 5.5

1. (4 pts) Let \( f(x) = [\sin x + 1]^2 \). Find \( f'(\pi) \) and use your answer to find the equation of the tangent to the curve at \((\pi, 1)\).

\[
\begin{align*}
  f'(x) &= 2[\sin x + 1] \cos x \\
  f'\left(\pi\right) &= 2[\sin \pi + 1] \cos \pi = 2[0 + 1][-1] = -2
\end{align*}
\]

Equation of tangent at \((\pi, 1)\): The slope of the tangent to the curve at \((\pi, 1)\) is \( f'(\pi) \) so set, \( y = mx + b = -2x + b \). Since \((\pi, 1)\) is a point on the curve, we can substitute \( x = \pi, y = 1 \) into this equation to find \( b \). Then \( 1 = -2\pi + b \), so \( b = 1 + 2\pi \). Then equation of the tangent at \((\pi, 1)\) is:

\[
y = -2x + (1 + 2\pi).
\]

2. (4 pts) Integrals

i) Evaluate: \( \int_0^1 \frac{t}{(t^2+1)^2} dt \)

Let \( u = t^2 + 1 \). Then \( du = 2t dt \). So, \( \int_0^1 \frac{t}{(t^2+1)^2} dt = 1/2 \int_1^2 \frac{1}{u^2} du = 1/2[-u^{-1}]_1^2 = 1/4 \)

Note the change in the integration limits when we use the substitution, \( u = t^2 + 1 \). When \( t = 0, u = 1 \), and when \( t = 1, u = 2 \).

ii) If \( F(x) = \int_0^x \frac{t}{(t^2+1)^2} dt \), what is \( F'(x) \)?

Since \( f(t) = \frac{t}{(t^2+1)^2} \) is continuous for all real numbers, by the First Fundamental Theorem of Calculus, \( F'(x) = f(x) = \frac{x}{(x^2+1)^2} \).

3. (2 pts) Let \( h(x) = f(x) - g(x) \) where \( f \) and \( g \) are differentiable functions on the real numbers. Suppose \( f(1) = g(1) \), and that \( f'(x) > g'(x) \) for all \( x \) in \([1, 10] \). Is \( h(x) \) positive on the open interval \((1,10)\)? Choose one of the following:

Since \( f(1) = g(1), h(1) = f(1) - g(1) = 0 \). Also, since \( f'(x) > g'(x) \) on \([1, 10] \), \( h'(x) = f'(x) - g'(x) > 0 \) on \([1, 10] \) and so is strictly increasing on that interval. If \( h(1) = 0 \), then \( h(x) > 0 \) for all \( x \) in \((1, 10) \). So the answer is: Yes.