MATH 1260 - Quiz 7
Solution

(1) Determine whether \((0, 0)\) is a maximum, minimum, or saddle point for \(f(x, y) = y^2\).

The second partial derivatives are

\[
\begin{pmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{pmatrix} = \begin{pmatrix}
  0 & 0 \\
  0 & 2
\end{pmatrix}.
\]

Since the determinant of this matrix is 0, the second derivative test is inconclusive.

However, it is clear that \((0, 0)\) is a minimum, though, since \(f(0, 0) = 0\), and \(y^2 \geq 0\) for every other value of \((x, y)\).
[NOTE: For this function, all of the points \((x, 0)\) are minimums].

(2) Find the critical points of \(f(x, y) = x^2 - y^2 + xy - 7\) and classify them as local maxima, minima, or neither.

The gradient of \(f\) is \(\nabla f(x, y) = (2x + y, -2y + x)\), and the only critical point is \((0, 0)\).

The second partial derivatives are

\[
\begin{pmatrix}
  f_{xx} & f_{xy} \\
  f_{yx} & f_{yy}
\end{pmatrix} = \begin{pmatrix}
  2 & 1 \\
  1 & -2
\end{pmatrix}.
\]

Since the determinant of this matrix is \(-5\), \((0, 0)\) is a saddle point.

(3) Find the extrema of \(f(x, y) = x^2 + y^2\) subject to the constraint \(x^4 + y^4 = 2\).

\(\nabla f = \lambda \nabla g\) if and only if either \(x = 0\) or \(y = 0\) or \(y = \pm x\). Plugging these into \(x^4 + y^4 = 2\), we obtain the following eight “special points:"

\((1, \pm 1), (-1, \pm 1), (\pm 4\sqrt{2}, 0), (0, \pm 4\sqrt{2})\).

The maximum is 2 at \((1, \pm 1), (-1, \pm 1),\) and the minimum is \(\sqrt{2}\) at \((\pm 4\sqrt{2}, 0), (0, \pm 4\sqrt{2})\).