(1) Compute the directional derivative of \( f(x, y) = x^2 + y^2 - 3xy^3 \) at the point \((1, 2)\) in the direction \( \vec{u} = \langle 1/2, \sqrt{3}/2 \rangle \).

\[
Df_{\vec{u}}(1, 2) = \nabla f(1, 2) \cdot \vec{u} = \langle -22, -32 \rangle \cdot \langle 1/2, \sqrt{3}/2 \rangle = -11 - 16\sqrt{3}.
\]

(2) Find the equation for the tangent plane to the surface \( x^2 + 2y^2 + 3z^2 = 10 \) at the point \((1, \sqrt{3}, 1)\).

\[
\nabla f(1, \sqrt{3}, 1) = \langle 2, 4\sqrt{3}, 6 \rangle,
\]

and the tangent plane is

\[
2x + 4\sqrt{3}y + 6z = 20.
\]

(3) Let \( y \) be a function of \( x \) satisfying \( F(x, y, x + y) = 0 \), where \( F(x, y, z) \) is a given function. Find a formula for \( \frac{dy}{dx} \).

Let \( f: \mathbb{R}^2 \to \mathbb{R} \) be the function \( f(x, y) = F(x, y, x + y) \). Then we know that \( \frac{dy}{dx} = -\frac{f_x}{f_y} \).

Consider the function \( g: \mathbb{R}^2 \to \mathbb{R}^3 \) defined by \( g(x, y) = (x, y, x + y) \). Then \( f \) is the composition of \( g \) and \( F \), i.e., \( f(x, y) = F(g(x, y)) \).

The chain rule tells us that

\[
Df = DF \cdot Dg,
\]

and therefore

\[
(f_x f_y) = (F_x F_y F_z) \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}.
\]

By working out the multiplication on the right we see that

\[
f_x = F_x + F_z, \quad f_y = F_y + F_z.
\]

Therefore,

\[
\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{F_x + F_z}{F_y + F_z}.
\]