(1) For each of the following, determine the type of conic:
   (a) $x^2 + xy + y^2 = 4$.
       This is an ellipse (Note: It is NOT a circle!).
   (b) $3x^2 + 3y^2 - 2xy - 6\sqrt{2}x - 6\sqrt{2}y = 8$.
       This is an ellipse.

(2) Show that if the acceleration of an object is always perpendicular to the velocity, then the speed of the object is constant.

Since the acceleration $\vec{a}(t)$ is always perpendicular to the velocity $\vec{v}(t)$, we have that their dot product is 0 for all $t$: $\vec{a}(t) \cdot \vec{v}(t) = 0$. Consider now the speed of the object $||\vec{v}(t)|| = \sqrt{\vec{v}(t) \cdot \vec{v}(t)}$. The derivative with respect to time of the speed squared is

$$\frac{d(\vec{v}(t) \cdot \vec{v}(t))}{dt} = 2\vec{v}(t) \cdot \frac{d\vec{v}(t)}{dt} = 2\vec{v}(t) \cdot \vec{a}(t) = 0,$$

and therefore the speed squared (and so also the speed itself) is a constant.

(3) Suppose that a particle follows the path $(e^t, e^{-t}, \cos t)$ until it flies off on a tangent at $t = 1$. Where is it at $t = 2$?

The point where it flies off is $\vec{r}(1) = (e, e^{-1}, \cos 1)$, and the tangent vector at that point is $\vec{v}(1) = (e, -e^{-1}, -\sin 1)$. A second later along the tangent line, the point is at

$$(e + e, e^{-1} - e^{-1}, \cos 1 - \sin 1) = (2e, 0, \cos 1 - \sin 1).$$