(1) Let $\vec{u} = \langle a, b \rangle$ be a vector. Find the projection $\vec{P}$ of $\vec{u}$ in the direction of $\langle 1, 1 \rangle$ in terms of $a$ and $b$. Let $\vec{N} = \vec{u} - \vec{P}$: Show that $\vec{N}$ is perpendicular to $\vec{P}$. 

Note: Each problem is worth 10 points. For full credit, explain your work and justify your answers.
(2) Calculate the following limits:

(a) \[ \lim_{(x,y) \to (0,0)} \frac{2x^2y}{x^4 + y^3}. \]

(b) \[ \lim_{(x,y) \to (0,0)} \frac{x^3 - y^3}{x^2 + y^2}. \]
(3) Find the plane passing through the point \((0, 1, 0)\) and perpendicular to the two following planes:

- The tangent plane to \(f(x, y) = \sin(xy) + 3xy^2 - 2e^{xy}\) at \((0, 1)\).
- The plane with normal vector \(\langle 1, 1, 1 \rangle\) passing through the point \((1, 1, 1)\).
(4) Let $f(x, y, z) = \left( x^2 + y^2 + z^2, \frac{9x}{x + y + z} \right)$ and $g(x, y) = (y - x, 2xy^2)$. Find the derivative of the composition $g \circ f$ at the point $(1, 1, 1)$. 
(5) Let \( f(x, y) = xy^2 - 5x^2y \). Find the maximum and minimum value of \( f \) on the region above the graph of \( y = x^2 \) and below the line \( y = 4 \).
(6) Find the maximum volume of a rectangular box with no top that can be built with 1200 square feet of material.
(7) Calculate the double integral \( \iint_R y^2 \, dA \) where \( R \) is the triangle with vertices \((2, 0), (0, 1), \) and \((1, 1)\).
(8) Set up the triple integral \( \iiint_K f(x, y, z) \, dV \) where \( K \) is the region below the graph of the function \( 4 - x^2 - y^2 \) and above the \( x, y \)-plane.
(9) Let $C$ be the circle of radius 2 and center $(0, 1, 0)$ in the $y, z$-plane. Show that

$$\int_C 4xy \, dx + e^{xyz} \, dx - y^2 \, dx - 2x^2 \, dy + y \, dz - x \, dz = \int_C 2xy^2 \, dx + x \, dx + 2xy \, dy - 2x^2 \, dy - z \, dy.$$
(10) Let $f(x, y, z)$ be a function. Calculate $\text{div}(\text{grad}(f))$ in terms of $f$ and its partial derivatives.
(Extra-credit) Let $A$ be the 3 by 3 matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$ 

(a) Find all the numbers $a$ such that $A\vec{v} = a\vec{v}$ for some non-zero vectors $\vec{v}$. [Hint: There are three such numbers $a$]

(b) For each $a$ that you found in part (a), find a non-zero vector $\vec{v}$ such that $A\vec{v} = a\vec{v}$. 

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