(1) Let \( f(x) = x^2 + 5 \). Using only the definition of derivative, calculate \( f'(3) \).

(2) Prove that the function \( f: \mathbb{R} \rightarrow \mathbb{R} \) defined by
\[
f(x) = \begin{cases} 
  x & x \neq 0 \\
  \frac{1}{1 + e^{1/x}} & x = 0
\end{cases}
\]
is continuous at 0 but not differentiable at 0.

(3) Prove that \( x^2 > x \ln x \) for all \( x > 0 \).

(4) Let \( f \) be differentiable on \([a, b]\). Suppose that there exists a point \( x_0 \in [a, b] \) such that \( f(x_0) > f(a) \) and \( f(x_0) > f(b) \). Prove that there exists two points \( c_1, c_2 \in (a, b) \) such that \( f'(c_1) < 0 < f'(c_2) \).

(5) Let \( f \) be differentiable on \([a, b]\), and assume that \( f(a) = f(b) \). True/False: If there exists a unique point \( c \in (a, b) \) such that \( f'(c) = 0 \), then \( c \) is a global extremum (i.e., either \( c \) is a global maximum or a global minimum, i.e., either \( f(c) \geq f(x) \) for all \( x \in [a, b] \) or \( f(c) \leq f(x) \) for all \( x \in [a, b] \)).

[If true, give a proof. If false, provide a counterexample.]

(6) Show that the inverse \( f^{-1} \) of \( f(x) = 3x^2 \sin x \) exists on the interval \((0, \pi/2)\) and calculate \( (f^{-1})'(3a^2 \sin 2) \).

(7) Let \( f: [a, b] \rightarrow \mathbb{R} \) be a function which is continuous everywhere on \([a, b]\) except at one point \( x_0 \in (a, b) \). Prove that \( f \) is integrable.

(8) Calculate
\[
\int_0^3 f(x) \, dx,
\]
where
\[
f(x) = \begin{cases} 
  x + 1 & 0 \leq x \leq 2 \\
  x^2 - 1 & 2 \leq x \leq 3
\end{cases}
\]
(9) Give an example of a function $f$ such that $f$ is integrable on $[a, b]$ but

$$F(x) = \int_a^x f(t)dt$$

is NOT everywhere differentiable on $[a, b]$.

(10) Use the Mean Value Theorem for integrals to prove that, if $f$ is integrable on $[0, 1]$, then

$$\lim_{n \to \infty} \int_0^1 x^n f(x)dx = 0.$$  

(11) Calculate the improper integral

$$\int_1^\infty \frac{1 + x}{x^3}dx.$$  

(12) Show that the product of two improperly integrable functions might not be improperly integrable.