Math 3210: Preparation for Test # 3
TRUE / FALSE QUESTIONS

For each of the following statements, either prove it (if it is true) or give an example to show that it is false. You are allowed to use any of the Theorems listed on the Test Web Page without having to prove them.

(1) True/False: If \( f \) is differentiable at \( a \), then
\[
\lim_{h \to 0} \frac{f(a + h) - f(a - h)}{2h} = f'(a).
\]

(2) True/False: Let \( f : \mathbb{R} \to \mathbb{R} \) be a function. If \( f'(x) = 0 \) for all \( x \in [a, b] \), then there exists a number \( C \) such that \( f(x) = C \) for all \( x \in [a, b] \).

(3) True/False: Let \( f : \mathbb{R} \to \mathbb{R} \) be a function. If \( f'(x) = 0 \) for all \( x \in (a, b) \), then there exists a number \( C \) such that \( f(x) = C \) for all \( x \in [a, b] \).

(4) True/False: Let \( f, g : \mathbb{R} \to \mathbb{R} \) be two functions. If \( f(x) \geq g(x) \) for all \( x \in \mathbb{R} \), then \( f'(x) \geq g'(x) \) for all \( x \in \mathbb{R} \).

(5) True/False: Let \( f, g : \mathbb{R} \to \mathbb{R} \) be two functions. If \( f'(x) \geq g'(x) \) for all \( x \in \mathbb{R} \), then \( f(x) \geq g(x) \) for all \( x \in \mathbb{R} \).

(6) True/False: On \([a, b]\), \( f(x) \) is integrable if and only if \(|f(x)|\) is integrable.

(7) True/False: On \([a, b]\), \( f(x) \) is integrable if and only if \( xf(x) \) is integrable.

(8) True/False: If \( f \) is differentiable on \([a, b]\), then
\[
\int_{a}^{b} f'(x) \, dx = f(b) - f(a).
\]

(9) True/False: If \( f \) is continuous on \((a, b)\), then \( f \) is improperly integrable on \((a, b)\) if and only if the following two limits exist and are finite:
\[
\lim_{x \to a^+} f(x), \quad \lim_{x \to b^-} f(x).
\]