MATH 2210
Test 3 Sample Questions
Answers (and Solutions of Selected Problems)

(1) Since \( \vec{r}(t) = <-t, 2-t> \) with \( 0 \leq t \leq 1 \), the integral is
\[
\int_0^1 (-t + (2 - t))\sqrt{2} \, dt = \sqrt{2}(-t^2 + 2t)|_0^1 = \sqrt{2}.
\]

(2) This is the gradient of \( f(x, y) = x^2y - y^2/2 \), and therefore the integral is equal to \( f(0, 3) - f(1, 1) = -5 \).

(3) The work is given by
\[
\int_C 2xy^2 \, dx + (2x^2y + z) \, dy + (y + x^2) \, dz.
\]
The integral
\[
\int_C 2xy^2 \, dx + (2x^2y + z) \, dy + y \, dz
\]
is 0 because this is the gradient of \( x^2y^2 + yz \) and \( C \) is a closed curve, and therefore the work is equal to
\[
\int_C x^2 \, dz.
\]
To calculate this integral, we need to parametrize each of the for sides of the square.
From \((0, 1, 0)\) to \((1, 1, 0)\): Here \( z = 0 \), \( dz = 0 \), and therefore \( \int_C x^2 \, dz = 0 \).
From \((1, 1, 0)\) to \((1, 1, 1)\): Here \( x = 1 \) and \( z = t \) with \( 0 \leq t \leq 1 \), and therefore \( \int_C x^2 \, dz = \int_0^1 dt = 1 \).
From \((1, 1, 1)\) to \((0, 1, 1)\): Here \( z = 1 \), \( dz = 0 \), and therefore \( \int_C x^2 \, dz = 0 \).
From \((0, 1, 1)\) to \((0, 1, 0)\): Here \( x = 0 \), and therefore \( \int_C x^2 \, dz = 0 \).
Therefore, the work is equal to 1.

(4) The flow is equal to
\[
\int_C -(x + y) \, dx + (x + y) \, dy.
\]
Since \( \vec{r}(t) = <t, t^2> \) with \( 0 \leq t \leq 2 \), we obtain
\[
\int_0^2 (-t + t^2)(1) + (t + t^2)(2t) \, dt = \int_0^2 (2t^3 + t^2 - t) \, dt = \left( \frac{t^4}{2} + \frac{t^3}{3} - \frac{t^2}{2} \right)|_0^2 = \frac{26}{3}.
\]

(5) The integral
\[
\int_C 5x \, dx + 6x^2z \, dx + y^2 \, dy + 2yz \, dy + y^2 \, dz + 2x^3 \, dz
\]
is 0 because it is the gradient of \( f(x, y, z) = 5x^2/2 + 2x^3z + y^3/3 + y^2z \) and \( C \) is a closed curve. Therefore, the integral is equal to \( \int_C \sin(x^2y) \, dz \). This is also 0 because \( z = 2 \) everywhere on \( C \), and therefore \( dz = 0 \).
(6) If $\mathbf{F} = \langle P, Q, R \rangle$, $\text{div}(\text{curl}(\mathbf{F})) = \text{div}(\langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle) = (R_{yx} - Q_{zx}) + (P_{zy} - R_{xy}) + (Q_{xz} - P_{yz})$, which is 0 because $P_{yz} = P_{zy}$, $Q_{xz} = Q_{zx}$, and $R_{xy} = R_{yx}$.

(7) $\text{curl}(\text{grad}(f)) = \text{curl}(\langle f_x, f_y, f_z \rangle) = \langle f_{zy} - f_{yz}, f_{xz} - f_{zx}, f_{yx} - f_{xy} \rangle$, which is $< 0, 0, 0 >$ because $f_{yz} = f_{zy}$, $f_{xz} = f_{zx}$, and $f_{yx} = f_{xy}$.

(8) The integral is $4\pi$.

(10) $\text{div}(\text{grad}(f)) = f_{xx} + f_{yy} + f_{zz}$. 