MATH 2210 - Limits of Multivariable Functions

Summary: Give a function \( f(x, y) \), we want to calculate the limit
\[
\lim_{(x, y) \to (0, 0)} f(x, y).
\]

- We have studied two ways to show that the limit does not exist:
  - Find two lines for which the limits of \( f(x, y) \) along those two lines exist and they are different from each other.
  - If all the lines give you the same limit, substitute a function of \( x \) for \( y \) (or a function of \( y \) for \( x \)) in order to obtain a limit which is different from the limits along lines.

- To show that the limit does exit, we have also studied two ways:
  - Find a function \( g(x, y) \) such that
    \[
    0 \leq |f(x, y)| \leq |g(x, y)| \quad \text{for every } x, y,
    \]
    and show that
    \[
    \lim_{(x, y) \to (0, 0)} g(x, y) = 0.
    \]
  - Change \( f(x, y) \) into polar coordinates, and show that the limit as \( r \to 0 \) exists no matter what \( \theta \) is.

Calculate the following limits or show that they do not exist:

\[
\begin{align*}
\lim_{(x, y) \to (0, 0)} \frac{x + 2y^2}{3x^2 + 2y^2} & \quad \lim_{(x, y) \to (0, 0)} \frac{2x^3y^2}{x^6 + y^4} \\
\lim_{(x, y) \to (0, 0)} \frac{xy}{\sqrt{3x^2 + 2y^2}} & \quad \lim_{(x, y) \to (0, 0)} \frac{x^2y}{3x^2 + 2y^2} \\
\lim_{(x, y) \to (0, 0)} \frac{x^2y}{x^4 + 3y^2} & \quad \lim_{(x, y) \to (0, 0)} \frac{3x^3 + 2\sqrt{y}}{x^2 + y^2} \\
\lim_{(x, y) \to (0, 0)} \frac{-3x^3 - y^2}{3x^3 + 2y^2} & \quad \lim_{(x, y) \to (0, 0)} \frac{x^4 + y^4}{x^2 + 3y^2} \\
\lim_{(x, y) \to (0, 0)} \frac{x^2y^5}{2x^4 + 3y^{10}} & \quad \lim_{(x, y) \to (0, 0)} \frac{x^2y^2}{2x^2 + 3y^4}
\end{align*}
\]