MATH 2210 - Test 3 Sample Questions (First Part)

(1) Find all the stationary points of the following functions, and classify them:

\[ f(x, y) = 5x^2 - 3xy - 2, \quad f(x, y) = 4xy + \ln(xy), \]
\[ f(x, y) = 5xy^2 - 3\sqrt{xy}, \quad f(x, y) = \frac{x + \sin(y^3)}{ey}. \]

(2) Find the global maximum and minimum of the following functions on the given region \( \mathcal{D} \):

(a) \( f(x, y) = x^2y - 3\sqrt{y} \) on \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1 \} \).
(b) \( f(x, y) = x^3 + xy - 3y^3 \) on \( \mathcal{D} \) is the triangle with vertices \((0, 0), (1, 0), \) and \((2, 2)\).
(c) \( f(x, y) = xy^2 - 1 \) on \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \} \).
(d) \( f(x, y) = x^2y^2 - 2xy \) on \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq x \} \).

(3) Calculate the following integrals on the given region \( \mathcal{D} \):

(a) \( \iint_{\mathcal{D}} xy \, dA \) where \( \mathcal{D} \) is the triangle with vertices \((0, 0), (0, 2), \) and \((1, 1)\).
(b) \( \iint_{\mathcal{D}} x \, dA \) where \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \} \).
(c) \( \iint_{\mathcal{D}} (x - y) \, dA \) where \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x, y \leq 1 \} \).
(d) \( \iint_{\mathcal{D}} (x^2 + y^2) \, dA \) where \( \mathcal{D} = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 2, 0 \leq y \leq x + 1 \} \).

(4) Find the mass and the center of mass of the triangle with vertices \((0, 1), (1, 0), \) and \((2, 2)\) with density \( \delta(x, y) \) given by the distance from the \( x \)-axis.

(5) Calculate the following integrals on the given region \( \mathcal{D} \):

(a) \( \iiint_{\mathcal{D}} (x + y + z) \, dV \) where \( \mathcal{D} \) is the cube \( 0 \leq x, y, z \leq 1 \).
(b) \( \iiint_{\mathcal{D}} x^2 \, dV \) where \( \mathcal{D} \) is the cube \( 0 \leq x, y, z \leq 2 \).

(6) Set up the following triple integrals as iterated integrals:

(a) \( \iiint_{\mathcal{D}} f(x, y, z) \, dV \) where \( \mathcal{D} \) is the tetrahedron with vertices \((0, 0, 0), (2, 0, 0), (0, 2, 0), \) and \((0, 0, 3)\).
(b) \( \iiint_{\mathcal{D}} f(x, y, z) \, dV \) where \( \mathcal{D} \) is the region in \( \mathbb{R}^3 \) delimited by the planes \( z = 0, x = 0, x = y, \) and \( x + y + z = 2 \).
(c) \( \iiint_{\mathcal{D}} f(x, y, z) \, dV \) where \( \mathcal{D} = \{(x, y, z) \in \mathbb{R}^3 \mid x, y \geq 0, 0 \leq z \leq 1 - x^2 + y^2 \} \).

(7) Find the mass and center of mass of the tetrahedron with vertices \((0, 0, 0), (1, 0, 0), (0, 1, 0), \) and \((0, 0, 1)\), with density \( \delta(x, y) = z \).

(8) Let \( f(x) \) be a function of the one variable \( x \), let \( f'(x) \) be its derivative, and let \( \mathcal{D} \) be the triangle with vertices \((0, 0), (1, 0), \) and \((0, 1)\). Show that

\[
\iint_{\mathcal{D}} f'(x) \, dA = \int_0^1 f(x) \, dx - f(0).
\]