MATH 2210 - Test 2 Sample Questions (First Part)

(1) For each of the following functions, describe the three cross-sections of the graph and sketch the level curves:

\( f(x, y) = x^2 - 3xy + 2y^2 - 2x + 3y \), \( f(x, y) = xy^2 \),

\( f(x, y) = -2x^2 + xy + y^2 - 5 \), \( f(x, y) = e^{xy} \).

(2) Calculate the following limits:

\[
\lim_{(x,y) \to (0,0)} \frac{3x^2y}{2x^4 - 3y^2}, \quad \lim_{(x,y) \to (0,0)} \frac{x^2 + y^2}{\sqrt{2x^2 + 3y^2}},
\]

\[
\lim_{(x,y) \to (0,0)} \frac{x^2 ye^y}{x^2 + y^1}, \quad \lim_{(x,y) \to (0,0)} \frac{y^2 + x^2 \sin(y)}{x^4 + 3y^2}.
\]

(3) Find the partial derivatives \( f_x \) and \( f_y \) of the following functions:

\( f(x, y) = 2x \sin(y) - 2x^2y \), \( f(x, y) = \ln(xy) + 3\sqrt{x} \),

\( f(x, y) = 2^x \sin(y) \), \( f(x, y) = \frac{3x^2y^2 - 2\sin(x)}{e^{xy}} \).

(4) Consider the functions

\[ f: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \quad f(x_1, x_2, x_3) = \left( x_1^2 x_3, 3x_1 + \frac{x_2}{x_3} \right) \]

\[ g: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad g(y_1, y_2) = \left( \sqrt{y_1}, 3y_1^2 - 2y_1 e^{y_1 y_2} \right). \]

Find the derivative of \( g \circ f \) at the point \((1, -3, 1)\).

(5) Let \( f(x, y) = x^2 - 2xy + 3y^2 - 2x + 5 \).

(a) Sketch the level curves.

(b) Find the gradient \( \nabla f \) at the point \((0, 0)\).

(c) Find all of the unit vectors \( \vec{v} \) such that \( Df_{\vec{v}}(0, 0) = 0 \).

(6) Let \( f: \mathbb{R}^2 \rightarrow \mathbb{R} \) be a function, let \( P \) be a point in \( \mathbb{R}^2 \), and let \( \nabla f(P) \) be the gradient of \( f \) at the point \( P \). Show that, if \( Df_{\nabla f(P)}(P) = 0 \), then \( Df_{\vec{v}}(P) = 0 \) for every vector \( \vec{v} \).