Quiz 4
Math 1040–1
June 22, 2012

Directions: Show all work for full credit. Clearly indicate all answers. Simplify all mathematical expressions completely. Unless otherwise directed, give each decimal approximation rounded to at least three decimal places.

Formulas:

\[ nP_r = \frac{n!}{(n-r)!} \]

Ways to order \( n \) objects with \( n_1 \) alike, \( n_2 \) alike, \ldots, and \( n_k \) alike = \( \frac{n!}{n_1!n_2!\cdots n_k!} \)

\[ nC_r = \frac{n!}{(n-r)!r!} \]

\[ \sigma^2 = \sum (x - \mu)^2 P(x) \]

1. A company that makes cartons finds that the probability of producing a carton with a puncture is 0.05, the probability that a carton has a smashed corner is 0.08, and the probability that a carton has a puncture and has a smashed corner is 0.004. Are the events “selecting a carton with a puncture” and “selecting a carton with a smashed corner” mutually exclusive? Find the probability that a carton chosen at random has a puncture or a smashed corner. (#15 from 3.3)

The events are not mutually exclusive, since it is possible for them to both happen at once.

\[ P(\text{puncture or smashed corner}) = P(\text{puncture}) + P(\text{smashed corner}) - P(\text{puncture and smashed corner}) \]

\[ = 0.05 + 0.08 - 0.004 = 0.126 \]

2. In a state lottery, you choose 5 different numbers out of 40. To win the top prize, your numbers must match the 5 numbers chosen by the lottery in any order. You purchase one lottery ticket. What is the probability that you will win the top prize? (#53 from 3.4)

Since numbers can be picked in any order, there are \( \binom{40}{5} = \frac{40!}{35!5!} = 658008 \) ways to choose 5 numbers at a time out of 40. Since there is only one winning combination, the probability of selecting the winning combination is \( \frac{1}{658008} = 0.00000152 \).

3. Find the number of distinguishable ways that the letters in “statistics” can be arranged. (#27 from 3.4)

Of the 10 letters in the word, there are 3 s’s, 3 t’s, 1 a, 2 i’s, and 1 c, so the total number of distinct arrangements of the letters is \( \binom{10}{3,3,1,2,1} = 50400 \).
4. Let $x$ represent the amount of snow (in inches) that fell in Nome, Alaska, last winter. Determine whether $x$ is discrete or continuous. Explain your reasoning. (#19 from 4.1)

$x$ is continuous, since the amount of snow in a year can be any value in the interval from 0 to infinity. (Note: Saying that $x$ can take an infinite number of values $x$ is not sufficient, since the set of whole numbers is infinite, but is discrete.)

5. The number of televisions per household in a small town of 2600 households are: (#29 from 4.1)

<table>
<thead>
<tr>
<th>Televisions</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td>26</td>
<td>442</td>
<td>728</td>
<td>1404</td>
</tr>
</tbody>
</table>

(a) Let $x$ represent the number of televisions for a randomly selected household. Construct a probability distribution for the above data:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>$\frac{26}{2600} = 0.01$</td>
<td>$\frac{442}{2600} = 0.17$</td>
<td>$\frac{728}{2600} = 0.28$</td>
<td>$\frac{1404}{2600} = 0.54$</td>
</tr>
</tbody>
</table>

(b) Calculate the mean of the probability distribution.

$$
\mu = \sum xP(x) = 0 \cdot 0.01 + 1 \cdot 0.17 + 2 \cdot 0.28 + 3 \cdot 0.54 = 2.35
$$

(c) Calculate the standard deviation of the probability distribution.

$$
\sigma^2 = \sum (x - \mu)^2 P(x) = (0 - 2.35)^2 \cdot 0.01 + (1 - 2.35)^2 \cdot 0.17 + (2 - 2.35)^2 \cdot 0.28 + (3 - 2.35)^2 \cdot 0.54 = 0.6275
$$

$$
\sigma = \sqrt{0.6275} = 0.792
$$