1 Integrale

Given a derivative, \( f'(x) \), the process of finding the function \( f(x) \) is called \textit{antidifferentiation} or \textit{integration}. The most general function \( f(x) \) is called the \textit{integral} or \textit{indefinite integral} of \( f'(x) \), and we write \( \int f'(x) \, dx = f(x) \).

**Examples**

Find each of the following:

1. \( \int 4x^3 \, dx = x^4 + C \), where \( C \) represents a general constant.
2. \( \int x^6 \, dx = \frac{x^7}{7} + C \)

**Powers of \( x \) Formula:** \( \int x^n \, dx = \frac{x^{n+1}}{n+1} + C \) for \( n \neq -1 \).

**Examples**

Calculate and simplify each of the following:

1. \( \int x^2 \, dx = \frac{x^3}{3} + C \)
2. \( \int \sqrt{x} \, dx = \int x^{1/2} \, dx = \frac{2x^{3/2}}{3} + C \)
3. \( \int \frac{1}{x^2} \, dx = \int x^{-3} \, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C \)

2 Properties

There are many general properties of integrals. Let \( c \) and \( C \) represent constants:

1. \( \int cu(x) \, dx = c \int u(x) \, dx \)
2. \( \int (u(x) \pm \nu(x)) \, dx = \int u(x) \, dx \pm \int \nu(x) \, dx \)
3. \( \int 1 \, dx = x + C \)
4. \( \int 0 \, dx = C \)

**Examples**

1. \( \int (3 + 2x^2) \, dx = 3x + \frac{2}{3}x^3 + C \)
2. \( \int (4 - \frac{1}{\sqrt{x^2}}) \, dx = \int (4 - x^{-2/3}) \, dx = 4x - 3x^{1/3} + C \)
3. \( \int \left( \frac{3}{x^2} - \frac{16}{\sqrt{x}} \right) \, dx = \int (3x^{-9} - 16x^{-1/5}) \, dx = -\frac{3}{8}x^{-8} - 20x^{4/5} + C \)

4. The marginal revenue for a product is \( \overline{MR} = -0.5x + 60 \). Find \( R(x) \).

\[
R(x) = \int (-0.5x + 60) \, dx = -0.25x^2 + 60x + C.
\]

Using that \( R(0) = 0 \), we see that \( C = 0 \), so \( R(x) = -0.25x^2 + 60x \).