Section 4.5, Graphs of Sine and Cosine Functions

Homework: 4.5 #7–13 odds, 37–49 odds, 53

For our graphs, we will assume that the angle $x$ is given in radians.

1 Graphs of Sine and Cosine

Let $y = \sin x$. Then its graph is:

(The hash marks on the $x$-axis are in increments of $\pi/2$.) Also, since sine is $2\pi$-periodic, this pattern repeats.

Let $y = \cos x$. Then its graph is:

(The hash marks on the $x$-axis are in increments of $\pi/2$ again.) Cosine is also $2\pi$-periodic, this pattern repeats.

The book shows more detailed graphs on page 332.

Our goal for the rest of the section will be to graph functions of the form

\[ y = a \sin(bx - c) + d \]
\[ y = a \cos(bx - c) + d, \text{ where } a, b, c, \text{ and } d \text{ are real numbers} \]

We will focus on one additional number at a time.

If $y = a \sin x$ or $y = a \cos x$, we say that $|a|$ is the amplitude of $y$. Now, instead of having the graph go from $y = -1$ to $y = 1$, it will go from $y = -a$ to $y = a$.

Examples

1. Sketch the graph of $y = 4 \sin x$.
   The amplitude of the function is 4. The graph is:
2. Sketch the graph of $y = -\frac{1}{2} \cos x$.

The amplitude of the function is $\frac{1}{2}$. The negative sign will “flip” the graph upside down.

Now, we will focus on functions of the form $y = a \sin(bx)$ and $y = a \cos(bx)$. We will assume that $b > 0$ (If $b$ is negative, we can use that sine is an odd function and that cosine is an even function to rewrite it for $|b|$.)

The **period** of the graph changes to $\frac{2\pi}{b}$.

**Examples**

1. Sketch the graph of $y = 3 \sin 2x$.

   The amplitude is 3 and the period is $\frac{2\pi}{2} = \pi$.

2. Sketch the graph of $y = -2 \cos \frac{x}{2}$.

   The amplitude of the function is 2. The negative sign will “flip” the graph upside down. The period of the graph is $\frac{2\pi}{\frac{1}{2}} = 4\pi$. 
Now, we will focus on horizontal and vertical shifts, which come from including the $c$ and $d$ in $y = a \sin(bx - c) + d$ and $y = a \cos(bx - c) + d$. (The $d$ creates the vertical shift, and the $c$ creates the horizontal shift.)

**Examples**

1. Sketch the graph of $y = \cos(x + \pi) - 3$.
   - The amplitude is 1 and the period is $2\pi$. The $-3$ will shift the graph down 3 units. The $+\pi$ will cause the “pattern” for cosine to go from $-\pi$ to $\pi$ instead of from 0 to $2\pi$ (we get these numbers by solving $x + \pi = 0$ and $x + \pi = 2\pi$).

2. Sketch the graph of $y = -4 \sin(4x - \frac{\pi}{2}) + 5$.
   - The amplitude is 4 and the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The graph is flipped upside-down (due to the $-4$ before the sine). It is also shifted up 5, and the cycle that normally goes from 0 to $2\pi$ is repeated from $\pi/8$ to $5\pi/8$ (we get these numbers by solving $4x - \frac{\pi}{2} = 0$ and $4x - \frac{\pi}{2} = 2\pi$).