Directions: Open a new xterm window and make sure you are in the Math1170 directory. Open Maple by using the xmaple & command. Save the worksheet as laboratoryexercises1.mws. Complete the following exercises, saving your work periodically. When you have completed the exercises, print out a copy of your worksheet to submit with your laboratory report. Laboratory reports will be due by class time on January 23, 2007.

1. On using the solve and fsolve commands

(a) Create and use a plot of the function \( f(x) = x^7 - 14x^5 + 49x^3 - 36x \) on the domain \(-3.2 \leq x \leq 3.2\) to identify the number and approximate location of all solutions of the equation \( f(x) = 0 \).

Recall from the laboratory handout that the solve command can be used to solve a set of equations. Generally speaking, the command line

\[
> \text{solve}(f(x)=0,x);
\]

solves the equation \( f(x) = 0 \) for \( x \). This command uses algebraic tricks (like factoring) to find solutions of equations.

(b) Use the solve command to find solutions of the equation \( x^7 - 14x^5 + 49x^3 - 36x = 0 \). Do your Maple generated solution(s) agree with your observations from part (a)?

(c) Check that your solution from part (b) is sensible by using the factor command on the function \( f(x) = x^7 - 14x^5 + 49x^3 - 36x \) to algebraically find solutions of the equation \( f(x) = 0 \).

The fsolve command is new to you. The solve command algebraically finds solutions of equations (if possible), whereas the fsolve command numerically finds solutions of equations. The fsolve command uses a numerical method called the Method of Bisection; you will study this method in more detail in the coming weeks in lecture. The command line

\[
> \text{fsolve}(f(x)=0,x=a..b);
\]

uses the Method of Bisection to find a single solution \( x \) that satisfies \( f(x) = 0 \) and \( a \leq x \leq b \).

(d) Create and use a plot of the function \( g(x) = e^{-x^2} - e^{-1000(x-0.13)^2} - 0.2 \) on the domain \(-1.5 \leq x \leq 1.5\) to identify the number and approximate location of all solutions of the equation \( g(x) = 0 \).

The command line below shows how to input the general exponential function \( r(x) = e^x \).

\[
> r:=x->\exp(1)^x;
\]
Using the command line above as inspiration, input the function \( g(x) \) into Maple and then use the \texttt{plot} command to generate a plot of the function on the desired domain.

(e) Use the \texttt{solve} command to find solutions of the equation \( e^{-x^2} - e^{-1000(x-0.13)^2} - 0.2 = 0 \). Do your Maple generated solution(s) agree with your observations from part (d)?

(f) Use the \texttt{fsolve} command to find solutions of the equation \( e^{-x^2} - e^{-1000(x-0.13)^2} - 0.2 = 0 \). Do your Maple generated solution(s) agree with your observations from part (d)?

Hint: Use your observations from part (d) to input appropriate search domains \( a \leq x \leq b \) and verify there is indeed a solution on that domain.

(g) In retrospect, why do you think that the \texttt{fsolve} command is potentially more appropriate for solving \( g(x) = 0 \) than the \texttt{solve} command is?

2. Studying an important family of functions

Families of functions share a common functional form and often have defining characteristics. For example, the family of functions defined by \( l(x) = mx + b \) is a family of lines in which each member has slope \( m \) and \( y \)-intercept \( b \). In this exercise, you will study a second family that is commonly used in mathematical models of biological phenomenon.

(a) Identify the independent variable, dependent variable, and parameter for the family \( h(x) = \frac{x^2}{k^2+x^2} \).

(b) Create a plot for the family members
   i. \( h(x) \) with \( k = 1 \),
   ii. \( h(x) \) with \( k = 2 \),
   iii. \( h(x) \) with \( k = 5 \),
   iv. and \( h(x) \) with \( k = 10 \),
   each on the domain \( 0 \leq x \leq 20 \).

(c) Use all four plots to (1) describe the defining characteristics (i.e. general qualitative behavior) of this family of functions and (2) describe how \( k \) influences this behavior.