The synthetic geometry of the Weil-Petersson metric

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The last few years has seen many new results and broadening interest in the study of the Weil-Petersson metric on Teichmüller space and its metric completion. Improved estimates on the behavior of the metric near infinity has allowed for a deeper understanding of the behavior of geodesics than was previously available.

In this mini-course I will give an overview of the central background results and estimates needed, describe recent developments in the large scale and synthetic geometry of the Weil-Petersson metric, and describe an emerging conjectural picture of the behavior of geodesics. Topics will include:

1. Wolpert’s non-completeness and convexity theorems and Masur’s description of the Weil-Petersson completion.

2. The large-scale geometry of the Weil-Petersson metric and the combinatorial geometry of simple closed curves on surfaces.

3. The CAT(0) geometry of the Weil-Petersson completion and the non-refraction of geodesics at the frontier.

4. The Masur-Wolf theorem that the isometry group of the Weil-Petersson metric is the mapping class group.

5. The Weil-Petersson visual sphere: density of cusps, non-continuity of Mod(S), and an ending lamination conjecture for Weil-Petersson geodesics.


The idea of the course will be to give an introduction containing enough background to understand current open problems in the field, and to provide a starting point for attacking these problems. While background in hyperbolic geometry and Teichmüller theory will be very helpful, I will try to make the discussion as self-contained as possible.