Mathematics 2210 Calculus III

Final Examination December 10,11,2003

You may use graphing calculators and tables of Integrals. You MUST show enough work to convince me that you know how to do the problems.

Part I. Do FIVE (5) of these first 7 problems.

1. Find the distance of the point (2,1,1) from the plane given by the equation $x + y - z = 1$.

2. A particle moves in the plane as a function of time: $\mathbf{X}(t) = (2t + 1)\mathbf{I} + (t^2 - 2t + 3)\mathbf{J}$. Find the tangential and normal components of the acceleration.

3. Find the points where the ellipse $x^2 + 2xy + 10y^2 = 63$ has a horizontal tangent.

4. Let $f(x,y) = x^2 + xy + y^2$. A particle is moving through the plane so that its position at time $t$ is $\mathbf{X}(t) = \sin t \mathbf{I} + \cos t \mathbf{J}$. Find $df/dt$ when $t = \pi/3$.

5. Find a vector perpendicular to the line given by the symmetric equations

$$\frac{x - 1}{2} = \frac{y + 1}{4} = \frac{z}{5}$$

lying in the plane given by the equation $2x + y + z = 0$.

6. Find the maximum value of $3x^2 - y^2 + z$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$.

7. Let $R$ be the part of the unit sphere in the first octant. Suppose that it is filled with a material whose density at the point $(x,y,z)$ is given by $\delta(x,y,z) = xyz$. Find the total mass.

Part II. Do ALL three problems.

8. Given the vector field $\mathbf{F}(x,y) = (x^2 - y)\mathbf{I} + (y^2 - x)\mathbf{J}$,

   a) find a function $f$ whose gradient is $\mathbf{F}$.

   b) Calculate $\text{div } \mathbf{F}$.

9. Let $D$ be the region in the plane bounded by the circle $x^2 + y^2 = 9$. Let $C$ be the boundary of $D$ traversed counterclockwise. Find

$$\int_C y^2 dx + 2xy dy$$

10. Let $C$ be the curve given parametrically by

$$\mathbf{X}(t) = t\mathbf{I} + t^2 \mathbf{J} + t^3 \mathbf{K}$$

for $t$ running from 0 to 2. For the vector field $\mathbf{F} = x\mathbf{I} + y\mathbf{J} + 2x\mathbf{K}$, find $\int_C \mathbf{F} \cdot T ds$. 

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