1. Find the focus and vertex (or foci and vertices) of the conic given by the equation \( x^2 - 8x - 8y = 8 \).

**Answer.** Complete the square:

\[
\begin{align*}
x^2 - 8x + 16 - 8y & = 8 + 16 , \\
\text{or } (x - 4)^2 & = 8(y + 3) .
\end{align*}
\]

This is the equation of a parabola which opens upward, and whose vertex is at \((4, -3)\). Since \(4p = 8\), \(p = 2\), so the focus is 2 units above the vertex, at \((4, -1)\).

2. Find the equation of the conic which has a focus at \((6, 2)\) and ends of the minor axis at \((1,7)\) and \((1,-3)\).

**Answer.** The center of the conic is midway between the ends of the minor axis, so is at \(C : (1,2)\). Thus the axes are the lines \(x = 1\), \(y = 2\), and \(b = 5\). Since a focus is at \((6,2)\), \(c = 6 - 1 = 5\). Thus \(a^2 = b^2 + c^2 = 25 + 25\), so \(a = 5\sqrt{2}\). Since the center is at \((1,2)\), and \(a^2 = 50\), \(b^2 = 25\), we have the equation

\[
\frac{(x - 1)^2}{50} + \frac{(y - 2)^2}{25} = 1 .
\]

If you first thought the conic might be a hyperbola, and tried \(c^2 = a^2 + b^2\) first, you would obtain \(a = 0\), so that excludes that possibility.

3. Find the equation of the tangent line of the hyperbola

\[
\frac{x^2}{4} - y^2 = 1
\]

at the point \((4, \sqrt{3})\).

**Answer.** Taking differentials, we obtain

\[
\frac{xdx}{2} - 2ydy = 0 .
\]

Putting in the values \(x = 4\), \(y = \sqrt{3}\) gives us \(2dx - 2\sqrt{3}dy = 0\), so the tangent line has slope \(dy/dx = 1/\sqrt{3}\). The equation thus is

\[
y - \sqrt{3} = \frac{x - 4}{\sqrt{3}} .
\]

4. Find the area of the region that lies outside the circle \(r = 1\) and inside the circle \(r = 2 \cos \theta\).

**Answer.** At the point of intersection of the two curves we have \(1 = 2 \cos \theta\), so \(\theta = \pm \pi/3\). Since the area inside a curve \(r = r(\theta)\) is given by integrating \(dA = (1/2)r^2 d\theta\), the area between the curves \(r = 2 \cos \theta\) and \(r = 1\) is

\[
Area = \frac{1}{2} \int_{-\pi/3}^{\pi/3} [(2 \cos \theta)^2 - 1^2] d\theta
= \int_{-\pi/3}^{\pi/3} (1 + 2 \cos(2\theta) - 1/2) d\theta
= \sqrt{3}
\]
5. Find the center, foci and vertices of the ellipse given in polar coordinates by the equation

\[ r = \frac{6}{1 + \frac{1}{2} \sin \theta} \]

**Answer.** This ellipse has a focus at the origin, and its vertices are at the points where \( r \) has a minimum and a maximum. The minimum is attained when \( \sin \theta \) is as large as it can be, so is at \( \theta = \pi/2 \), with \( r = 12 \). The maximum is at \( \theta = -\pi/2 \), with \( r = 4 \). Thus the y-axis is the major axis of the ellipse, and the vertices are at \((0,12), (0,-4)\). The center is the midpoint of this segment, so is at \((0,4)\). Thus \( c = 4 \) (the distance from the center to a focus, and \( a = 8 \), the distance from the center to a vertex. In summary:

Center : \((0,4)\)  Foci : \((0,0), (0,8)\)  Vertices : \((0,-4), (0,12)\).

For the record, since \( b^2 = a^2 - c^2 = 64 - 16 = 48 \), \( b = 4 \sqrt{3} \), and the equation of the ellipse in cartesian coordinates is

\[ \frac{x^2}{48} + \frac{(y-4)^2}{64} = 1 \]

An alternative method is to switch to cartesian coordinates first. We can rewrite the equation as \( r(1 + \sin \theta/2) = 6 \) or, in cartesian coordinates, \( \sqrt{x^2+y^2} + y/2 = 6 \). Moving the term \( y/2 \) to the right hand side and squaring we get

\[ x^2 + y^2 = 36 - 6y + \frac{y^2}{4} \]

which brings us to (1) after combining terms and completing the square.