Find all the integrals. Remember that definite integrals should have numerical answers. You MUST show your work.

1a. \[ \int (x+1)x^{12}dx = \int (x^{12} + x^{13})dx = \frac{x^{13}}{13} + \frac{x^{14}}{14} + C. \]

1b. \[ \int \ln(x^2)dx = \int 2\ln xdx. \]

Let \( u = \ln x, \, dv = dx, \, du = dx/x, \, v = x \) and integrate by parts:
\[
\int \ln(x^2)dx = 2\int \ln xdx = 2(x\ln x - \int dx) = 2(x\ln x - x + C). 
\]

2. \[ \int \frac{dx}{x^2(x+2)} \]

**Answer.** We want to use partial fractions, so we begin with
\[
\frac{1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} = \frac{Ax(x+2) + B(x+2) + Cx^2}{x^2(x+2)}. 
\]

Evaluate at the roots: for \( x = 0, \) we get \( 1 = 2B, \) so \( B = 1/2; \) for \( x = -2, \) \( 1 = 4C, \) so \( C = 1/4. \) TO find \( A \) we have to equate the coefficients of \( x^2: \) \( 0 = A + C, \) so \( A = -1/4. \) Thus, we get
\[
\frac{1}{x^2(x+2)} = \frac{-1/4}{x} + \frac{1/2}{x^2} + \frac{1/4}{x+2},
\]
so that
\[
\int \frac{dx}{x^2(x+2)} = -\frac{1}{4}\ln x - \frac{1}{2x} + \frac{1}{4}\ln(x+2) + C = -\frac{1}{2x} + \frac{1}{4}\ln\left(\frac{x+2}{x}\right) + C.
\]

3. \[ \int_0^2 \frac{x}{1 + x^4}dx \]

**Answer.** Recall that the integral of \((1+u^2)^{-1}du = \arctan u. \) So, substitute \( u = x^2, \, du = 2xdx: \)
\[
\int_0^2 \frac{x}{1 + x^4}dx = \frac{1}{2} \int_0^4 \frac{du}{1 + u^2} = \frac{1}{2} \arctan 4.
\]
4. \[ \int x \sin x \, dx \]

**Answer.** We want to integrate by parts, taking \( u = x, \ dv = \sin x, \ du = dx, \ v = -\cos x: \)

\[ \int x \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + C. \]

5. \[ \int_2^4 \frac{dx}{x(x - 1)} \]

**Answer.** Use partial fractions to find

\[ \frac{1}{x(x - 1)} = \frac{1}{x - 1} - \frac{1}{x}. \]

Then

\[ \int_2^4 \frac{dx}{x(x - 1)} = \int_2^4 \left( \frac{1}{x - 1} - \frac{1}{x} \right) \, dx = \ln \left( \frac{x - 1}{x} \right)^4 \mid_2^4 \]

\[ = \ln \frac{3}{4} - \ln \frac{1}{2} = \ln \frac{3}{2}. \]