Find all the integrals. Remember that definite integrals should have numerical answers.

1a. $\int x \ln(2x) \, dx$

**Answer.** Integrate by parts so that the logarithm disappears: let $u = \ln(2x)$, $du = dx/x$ (notice the cancellation of the 2’s), $dv = xdx$, $v = x^2/2$:

$$\int x \ln(2x) \, dx = \frac{x^2}{2} \ln(2x) - \frac{1}{2} \int x \, dx = \frac{x^2}{2} \ln(2x) - \frac{x^2}{4} + C .$$

1b. $\int \frac{\ln(2x)}{x} \, dx$

**Answer.** As we saw above, letting $u = \ln(2x)$, $du = dx/x$, we have

$$\int \frac{\ln(2x)}{x} \, dx = \int u \, du = \frac{u^2}{2} + C = \frac{\ln(2x)^2}{2} + C = \frac{\ln x}{2} + C .$$

2. $\int_2^4 \frac{dx}{x^2 - 1}$

**Answer.** We have the partial fractions expansion

$$\frac{1}{x^2 - 1} = \frac{1}{2} \left( \frac{1}{x-1} - \frac{1}{x+1} \right) ,$$

so

$$\int_2^4 \frac{dx}{x^2 - 1} = \frac{1}{2} \left( \ln(x-1) - \ln(x+1) \right) \bigg|_2^4 = \frac{1}{2} \left( \ln 3 - \ln 5 - (\ln 1 - \ln 3) \right) = \frac{1}{2} \ln \left( \frac{9}{5} \right) .$$

3. $\int \tan^2 x \, dx$

**Answer.** Alas, $\tan^2 x = \sec^2 x - 1$, so

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C .$$

4a. $\int e^x (e^{2x} + 1) \, dx$

**Answer.** $\int e^x (e^{2x} + 1) \, dx = \int (e^{3x} + e^x) \, dx = \frac{1}{3} e^{3x} + e^x + C .$

4b. $\int x (e^{2x} + 1) \, dx$

**Answer.** Here we must use integration by parts: $u = x$, $du = dx$, $dv = (e^{2x} + 1) \, dx$, $v = (1/2) e^{2x} + x$:

$$\int x (e^{2x} + 1) \, dx = x \left( \frac{1}{2} e^{2x} + x \right) - \int \left( \frac{1}{2} e^{2x} + x \right) \, dx = \frac{x}{2} e^{2x} + x^2 - \frac{1}{4} e^{2x} - \frac{x^2}{2} + C .$$
5. \( \int_1^2 \frac{dx}{x^2(x+1)} \)

**Answer.** We have a partial fractions expansion of the form

\[
\frac{1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}.
\]

Putting the expression on the right over the common denominator, we have equality of the numerators:

\[
1 = Ax^2 + Bx(x+1) + C(x+1).
\]

At \( x = -1 \) we get \( 1 = A \),

At \( x = 0 \) we get \( 1 = C \),

Coefficient of \( x^2 \): \( 0 = A + B \), so that \( B = -1 \).

Thus

\[
\int_1^2 \frac{dx}{x^2(x+1)} = \int_1^2 \frac{dx}{x+1} - \int_1^2 \frac{dx}{x} + \int_1^2 \frac{dx}{x^2}
\]

\[
= (\ln 3 - \ln 2) - (\ln 2 - \ln 1) - (\frac{1}{2} - 1) = \ln 3 - 2\ln 2 + \frac{1}{2} = \frac{1}{2} + \ln(\frac{3}{4}).
\]