1. Differentiate:

a) \( f(x) = e^x \ln(x^2) \)

\[
f'(x) = e^x \ln(x^2) + e^x \frac{2x}{x^2} = e^x (\ln(x^2) + 2x). \]

b) \( g(x) = e^{3\sin(2x)} \)

\[
g'(x) = e^{3\sin(2x)} (3\cos(2x))(2) = 6e^{3\sin(2x)} \cos(2x). \]

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2. Integrate:

a) \( \int \frac{xe^x}{e^x + 1} \, dx \)

Let \( u = e^x \), \( du = 2xe^x \, dx \), so that the integral becomes

\[
\frac{1}{2} \int \frac{du}{u+1} = \frac{1}{2} \ln(u+1) + C = \frac{1}{2} \ln(e^x + 1) + C. 
\]

b) \( \int_1^3 \frac{(\ln x)^2}{x} \, dx \)

Let \( u = \ln x \), \( du = dx/x \). When \( x = 1 \), \( u = 0 \) and for \( x = 3 \), \( u = \ln 3 \). The integral becomes

\[
\int_0^{\ln 3} u^2 \, du = \frac{u^3}{3} \bigg|_0^{\ln 3} = \frac{(\ln(3))^3}{3}. 
\]

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3. The Zombie National Bank offers accounts which pay 10.5% annually, compounded continuously. How much should I invest today so as to have $12,000 in 6 years?

The equation for continuous growth is \( P = P_0e^{rt} \). Here \( r = .105 \), \( t = 6 \), \( P = 12000 \), and we are to solve for \( P_0 \). We have

\[ \quad 12000 = P_0e^{105(6)} \quad \text{or} \quad P_0 = 12000e^{-105(6)} = 6391.10. \]

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4. A certain radioactive element decays so that in 100 years it has decreased to 82% its original size. What is its half-life?

Let \( T \) be the half-life of the element (in years), and \( r \) the annual rate of decay. We have the two equations

\[
.82 = e^{100r} \quad \text{or} \quad .5 = e^{rT}. 
\]

From the first equation \( r = \ln(.82)/100 \), and then the second equation becomes

\[
\ln(.5) = rT = \frac{\ln(.82)}{100} T
\]

\[
\]
giving the answer $T = 349.28$ years.

5. Solve the initial value problem $y' + y = e^x$, $y(0) = 5$.

First solve the homogeneous equation $y' + y = 0$. This has the solution $y = Ke^{-x}$. We try $y = ue^{-x}$ in the given equation, leading to

$$u'e^{-x} = e^x \quad \text{or} \quad u' = e^{2x}$$

which has the solution $u = e^{2x}/2 + C$. Thus the general solution of our equation is

$$y = \left(\frac{e^{2x}}{2} + C\right)e^{-x} = \frac{e^x}{2} + Ce^{-x}.$$  

The initial conditions are $y = 5$ when $x = 0$. Put that in the above equation and solve for $C$ to get $C = 9/2$. Thus the answer is

$$y = \frac{e^x + 9e^{-x}}{2}.$$