1. Differentiate:
   a) \( f(x) = e^x \ln x \)

   **Answer.** Use the product rule:
   \[
   f'(x) = e^x \ln x + \frac{e^x}{x}.
   \]

   b) \( g(x) = e^{2x^2 + 3x - 1} \)

   **Answer.** Use the chain rule:
   \[
   g'(x) = e^{2x^2 + 3x - 1}(4x + 3).
   \]

2. Integrate
   a) \( \int e^{\ln x + 1} \, dx \)

   **Answer.** By the rules of exponentials, \( e^{\ln x + 1} = e^{\ln x} e = xe \). Thus
   \[
   \int \ln(3e^x) \, dx = \int ex \, dx = e\frac{x^2}{2} + C.
   \]

   b) \( \int_0^3 e^{(3x + 1)} \, dx \)

   **Answer.**
   \[
   \int_0^3 (e^{3x} + e^x) \, dx = \left(\frac{e^{3x}}{3} + e^x\right)_0^3 = \frac{e^9}{3} + e - \frac{4}{3}.
   \]

3. I want to invest $5000 in a growth fund so that in 5 years I will have $8000. What interest rate, compounded continuously will produce that growth?

   **Answer.** The data give us the equation \( 8 = 5e^{5r} \), where \( r \) is the rate desired. Thus
   \[
   r = \frac{1}{5} \ln\left(\frac{8}{5}\right) = .094 \quad \text{or} \quad 9.4\%.
   \]

4. A certain radioactive element decays so that in 47 years it has decreased to 80% its original size. What is its half-life?

   **Answer.** Again, the decay equation is \( A(t) = A_0 e^{-rt} \), where \( r \) is the rate of decay, \( t \) is the time, \( A(t) \) is the amount at time \( t \), and \( A_0 \) is the amount at time \( t = 0 \). We are told that \( .8 = (1)e^{-(.094)} \), and we are asked to find the \( T \) such that \( .5 = e^{-rT} \). From the first equation we find
   \[
   -47r = \ln(.8), \quad \text{so that} \quad r = \frac{\ln(.8)}{-47} = 4.75 \times 10^{-3}.
   \]

   Then the half-life is the solution to \(.5 = e^{-4.75 \times 10^{-3}T}, \) so that
   \[
   T = \frac{\ln(2)}{4.75 \times 10^{-3}} = 146 \text{ years}.
   \]
5. Solve the initial value problem $xy' + y = x$, $y(2) = 5$.

**Answer.** First solve the homogeneous equation $xy' + y = 0$, for which the variables separate: $dy/y = -dx/x$. This integrates to $\ln y = -\ln x + C = \ln(1/x) + C$, which in turn exponentiates to $y = K/x$. So, we try $y = u/x$, $y' = u'/x + \cdots$ in the original equation, getting

$$xu'/x = x \quad \text{or} \quad u' = x,$$

which has the solution $u = x^2/2 + C$. Thus

$$y = \frac{u}{x} = \frac{x}{2} + \frac{C}{x}.$$

The initial condition gives $5 = 1 + C/2$, so $C = 8$, and the answer is

$$y = \frac{u}{x} = \frac{x}{2} + \frac{8}{x}.$$