1. (15 pts) Find the indicated derivatives. No need to simplify.
   (a) (5pts) $D_x(\ln(\tan(x)))$
   $$= \frac{1}{\tan x} \sec^2 x = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} = \frac{1}{\sin x \cos x}.$$  

   (b) (5pts) $D_x(e^{\sin(7x)})$
   $$= e^{\sin(7x)} \cos(7x) \cdot 7 = e^{\sin(7x)} \cdot 7 \cdot \cos(7x).$$  

   (c) (5pts) $D_x(x^{5x}) = D_x(e^{5x \ln x})$
   $$= e^{5x \ln x} \left( 5x \ln x + 5x \left( \frac{1}{x} \right) \right) = x^{5x} \left( 5 \ln x + 5 \right).$$

2. (10pts) Find the indicated antiderivatives. Remember +C!!
   (a) (5pts) $\int e^x \cosh(e^x) \, dx$
   $$u = e^x \quad du = e^x \, dx$$
   $$= \int \cosh(u) \, du = \sinh(u) + C = \sinh(e^x) + C$$

   (b) (5pts) $\int \frac{1}{2 - 3x} \, dx$
   $$u = 2 - 3x \quad du = -3 \, dx$$
   $$= -\frac{1}{3} \int \frac{1}{u} \, du = -\frac{1}{3} \ln \left| u \right| + C = -\frac{1}{3} \ln \left| 2 - 3x \right| + C.$$
3. (4pts) Suppose the function $f$ is one-to-one. Listed below are a few values of $f$ and its derivative:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

For example, $f(0) = 1$, $f'(1) = 4$, etc. Fill in the blanks:

2. $f^{-1}(2) = \frac{1}{3}$

2. $(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{4}$

4. (4pts) Find a formula for the inverse of the function $f(x) = x^2 + 2x + 5$, $x > -1$.

\[
x = y^2 + 2y + 5 \\
= (y^2 + 2y + 1) + 4 \\
= (y + 1)^2 + 4 \\
\]

5. (10pts) Evaluate the following. Any answer representing an angle should be given in radians.

2. (a) $\sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$

2. (b) $\cos^{-1}(0) = \frac{\pi}{2}$

2. (c) $\sin(\sin^{-1}(\frac{1}{2})) = \frac{1}{2}$

2. (d) $\sin^{-1}(\sin(3\pi)) = 0$

2. (e) $\cos(\sin^{-1}(\frac{\sqrt{3}}{2})) = \frac{\sqrt{3}}{2}$

6. (10pts) A 100 gram sample of a radioactive isotope was found to only contain 73 grams of radioactive material after 17 days. Let $A(t)$ denote the amount of radioactive material (in grams) after $t$ days. So $A(0) = 100$ and $A(17) = 73$.

(a) (6pts) Find constants $C$ and $k$ such that $A(t) = Ce^{kt}$. No need to simplify.

3. $C = 100$ (100 = $A(0) = Ce^{k \cdot 0} = C$)

3. $k = \frac{\ln(\frac{73}{100})}{17}$ (73 = $A(17) = 100 \cdot e^{k(17)}$)

(b) (4pts) Find the half-life of the isotope; that is, the amount of time (in days) that it takes for half of the isotope to decay. No need to simplify.

4. $\frac{1}{2} = e^{\frac{k}{17} \ln(\frac{73}{100})}$ (\ln(\frac{1}{2}) = \frac{17 \ln(\frac{73}{100})}{\ln(\frac{73}{100})} \Rightarrow t = \frac{17 \ln(\frac{1}{2})}{\ln(\frac{73}{100})}$
7. (25 pts) Evaluate the following indefinite integrals. Remember +C!!

(a) \( \int \cos^3 x \sin^2 x \, dx \)
\[
= \int \cos x \left( 1 - \sin^2 x \right) \sin^2 x \, dx = \int (u^2 - u^4) \, du
\]
\[
u = \cos x, \quad du = \cos x \, dx
\]
\[
= \frac{1}{3} u^3 - \frac{1}{5} u^5 + C = \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C
\]

(b) \( \int xe^{3x} \, dx \)
\[
= \frac{1}{3} xe^{3x} - \frac{1}{3} \int e^{3x} \, dx
\]
\[
u = e^{3x}, \quad dv = dx
\]
\[
= \frac{1}{3} xe^{3x} - \frac{1}{9} e^{3x} + C
\]

(c) \( \int \sin^{-1} x \, dx \) \quad Hint: Use integration by parts with \( u = \sin^{-1} x \) and \( dv = dx \).
\[
u = x \quad \frac{1}{\sqrt{1-x^2}} \quad du = \frac{1}{\sqrt{1-x^2}} \, dx
\]
\[
x \sin^{-1} x - \frac{1}{2} \int \frac{x}{\sqrt{1-x^2}} \, dx = x \sin^{-1} x + \frac{1}{2} \int u^{1/2} \, du
\]
\[
= x \sin^{-1} x + \frac{1}{3} u^{3/2} + C = x \sin^{-1} x + \sqrt{1-x^2} + C
\]

(d) \( \int \sec x \tan^3 x \, dx \)
\[
= \int \sec x \tan x \tan^2 x \, dx
\]
\[
= \int \sec x \tan x (\sec^2 x - 1) \, dx
\]
\[
u = \sec x, \quad dv = \sec x \tan x \, dx
\]
\[
= \int (u^2 - 1) \, du = \frac{1}{3} u^3 - u + C = \frac{1}{3} \sec^3 x - \sec x + C
\]

(e) \( \int \cos^2 x \sin^2 x \, dx \)
\[
= \frac{1}{4} \int (1 + \cos(2x)) (1 - \cos(2x)) \, dx = \frac{1}{4} \int (1 - \cos^2(2x)) \, dx
\]
\[
= \frac{1}{4} \int (1 - \frac{1}{2} (1 + \cos(4x))) \, dx
\]
\[
= \frac{1}{4} \int \left( \frac{1}{2} - \frac{1}{2} \cos(4x) \right) \, dx
\]
\[
= \frac{1}{8} x - \frac{1}{32} \sin(4x) + C
\]
8. (12pts) Evaluate the following indefinite integral using **rationalizing substitution** in three steps.

\[ \int \frac{\sqrt{1 - x^2}}{x^4} \, dx \]

Should be

\[ \int \frac{1 + x^2}{x^4} \, dx \]

(a) (4pts) First, write as an integral with respect to \( t \) by using the rationalizing substitution \( x = \tan t \).

\[
\begin{align*}
\sqrt{1 + x^2} &= \sec t \\
x &= \tan t \Rightarrow dx &= \sec^2 t \, dt \\
\int \frac{1 + x^2}{x^4} \, dx &= \int \frac{\sec^2 t}{\tan^4 t} \, dt
\end{align*}
\]

(b) (4pts) Second, evaluate the integral from part (a). You answer should be in terms of \( t \). **Hint:**

Write the integrand in terms of \( \sin t \) and \( \cos t \) using the identities \( \tan t = \frac{\sin t}{\cos t} \) and \( \sec t = \frac{1}{\cos t} \).

\[
\begin{align*}
&= \int \frac{\sec^2 t}{\tan^4 t} \, dt \\
&= \int \left( \frac{1}{\cos^2 t} \right) \left( \frac{\cos^4 t}{\sin^4 t} \right) \, dt \\
&= \int \frac{\cos^2 t}{\sin^4 t} \, dt \\
&= \int \frac{1}{\sin^2 t} \, du = \frac{1}{3} u^{-3} + C = \frac{1}{3} \sin^{-3} t + C
\end{align*}
\]

(c) (4pts) Finally, write you answer to part (b) in terms of \( x \).

\[
\begin{align*}
\frac{X}{1} &= \tan t \\
\sin t &= \frac{x}{\sqrt{1 + x^2}} \\
\frac{1}{3} \left( \frac{1}{\sin^3 t} \right) + C &= \frac{1}{3} \left( \frac{1}{X^3} \right) + C
\end{align*}
\]

9. (10pts) Use partial fractions to find \( \int \frac{7}{x^2 - 3x - 10} \, dx \).

\[
\frac{7}{x^2 - 3x - 10} = \frac{A}{x - 5} + \frac{B}{x + 2}
\]

plug in \( x = 5 \): \( 7 = 7A \Rightarrow A = 1 \).

plug in \( x = -2 \): \( 7 = -7B \Rightarrow B = -1 \).

\[
\int \frac{7}{x^2 - 3x - 10} \, dx = \int \left( \frac{1}{x - 5} - \frac{1}{x + 2} \right) \, dx
\]

\[
= \ln |x - 5| - \ln |x + 2| + C
\]