1. Find the equation of the line which goes through the point (2, -1) and is perpendicular to the line given by the equation $2x - y = 1$.

**Answer.** The given line has slope 2, so the line we seek has slope $-\frac{1}{2}$. (2, -1) lies on the line, so the equation is
\[\frac{y - (-1)}{x - 2} = -\frac{1}{2}\]
which simplifies to $y = -(1/2)x$.

2. a) Let $f(x) = x^2 + 3x - 1$. Find the slope of the line joining the points $(2, 9)$ and $(x, f(x))$.
b) Find the slope of the tangent line to the curve $y = f(x)$ at the point $(2, 9)$.
c) What is the equation of this tangent line?

**Answer.** a) The line joining these two points has slope
\[
\frac{x^2 + 3x - 1 - 9}{x - 2} = \frac{x^2 + 3x - 10}{x - 2} = x + 5
\]
by long division.
b). $f'(x) = 2x + 3$, so at $x = 2$, the tangent line has slope $f'(2) = 7$. (Notice that if we substitute $x = 2$ in the answer to part a), we get the same answer, confirming the statement after equation (1.11)).
\[
\frac{y - 9}{x - 2} = 7 \quad \text{or} \quad y = 7x - 5.
\]

3. Let $y = x^3 - 3x + 1$. Find the points on the curve whose tangent lines have slope $m = 9$.

**Answer.** The slope of the tangent line at $(x, y)$ is $dy/dx = 3x^2 - 3$. Set this equal to 9 and solve:
\[3x^2 - 3 = 9 \quad \text{or} \quad 3x^2 = 12
\]
which has the solutions $x = \pm 2$.

4. Find the derivatives of the following functions:
   a) $f(x) = x^3 - x^2 + 1$
   b) $g(x) = x^2 + \frac{1}{x^3}$
   c) $h(x) = (x^2 + 1)(x^3 - x^2 + 1)$

**Answer.** a) $f'(x) = 3x^2 - 2x$.

b) First write the function in exponential notation: $g(x) = x^2 + x^{-1}$. Then $g'(x) = 2x - 3x^{-4}$.

c) We use the product rule and the answers to parts a) and b):
\[h'(x) = (2x - 3x^{-4})(x^3 - x^2 + 1) + (x^2 + \frac{1}{x^3})(3x^2 - 2x)\]
5. Find the derivatives of the given functions:
   a) \( f(x) = 3x^{-1} + x^3 \)
   b) \( g(x) = (x^3 + 1)^4 \)
   c) \( h(x) = (\cos(2x) + 1) \sin(3x) \)

**Answer.**

a) \( f'(x) = -3x^{-2} + 3x^2 \).

b) \( g'(x) = 4(x^3 + 1)^3(3x^2) = 12x^2(x^3 + 1)^3 \).

c) Use the product rule and chain rules carefully:
\[
h'(x) = (\cos(2x) + 1)(\cos(3x))(3) + (-\sin(2x))(2)(\sin(3x)) \\
= 3(\cos(2x) + 1) \cos(3x) - 2\sin(2x) \sin(3x) \]

6. Find the derivative of
\[
f(x) = \frac{x^2 + 1}{x + 1} \]

**Answer.**

**7. Find the derivatives of the following functions:**
   a) \( f(x) = \cos^2x \)
   b) \( g(x) = \frac{\sin x}{\cos^2x} \)

**Answer.**

a) \( f'(x) = 2\cos x(-\sin x) = -2\cos x \sin x \).

\[
g'(x) = \frac{\cos^2 x \cos x - \sin x(-2\cos x \sin x)}{\cos^4 x} = \frac{\cos^3 x + 2\cos x \sin^2 x}{\cos^4 x} \\
= \frac{\cos^2 x + 2\sin^2 x}{\cos^3 x} = \frac{\cos^2 x}{\cos^3 x} + \frac{2\sin^2 x}{\cos^3 x} \]
which is where you would get if you first wrote \( g(x) = \tan x \sec x \), and then used the product rule.

8. Find the equation of the line tangent to the curve \( y = \cos(x/2) \) at \((3\pi,0)\)

**Answer.**

Take differentials: \( dy = -(1/2) \sin(x/2) \, dx \). At \( x = 3\pi \), we get \( dy = -(1/2)(-1) \, dx = dx/2 \). Thus the equation of the tangent line is \( y = (x - 3\pi)/2 \).

9. Let \( f(x) = x^3 - 8x^2 + 3 \). Find the interval in which \( f'(x) < 0 \).

**Answer.**

\( f'(x) = 3x^2 - 16x = x(3x - 16) \). This is zero when \( x = 0 \) and \( x = 16/3 \). \( f' \) is of constant sign in the intervals separated by these points, so we need only check for a particular point. Since \( f'(1) = -13 \), \( f'(x) < 0 \) for \( x \) in \((0, 16/3)\).

10. An object moves in a straight line so that its position at time \( t \) is given by \( x(t) = t(t^2 + 1)^2 \). What is the velocity of the object when \( t = 2? \)
\[ v = \frac{dx}{dt} = (t^2 + 1)^2 + t(2(t^2 + 1))(2t) = (t^2 + 1)(t^2 + 1 + 4t^2) = (t^2 + 1)(5t^2 + 1). \]

11. Let \( f(x) = (x - \sqrt{x})^2 \). Find \( f'(x) \) and \( f''(x) \).

**Answer.**

\[ f(x) = (x - \sqrt{x})^2 = x^2 - 2x\sqrt{x} + x = x^2 - 2x^{3/2} + x \]

\[ f'(x) = 2x - 3x^{1/2} + 1 \quad f''(x) = 2 - \frac{3}{2}x^{-1/2}. \]

12. Sketch the graph of a function with these properties:
   a) \( f(0) = 2 \) and \( f(1) = 0 \);
   b) \( f'(x) < 0 \) for \( 0 < x < 2 \);
   c) \( f''(x) > 0 \) for \( x < 0 \) or \( x > 2 \).

**Answer.**

![Graph of a function]