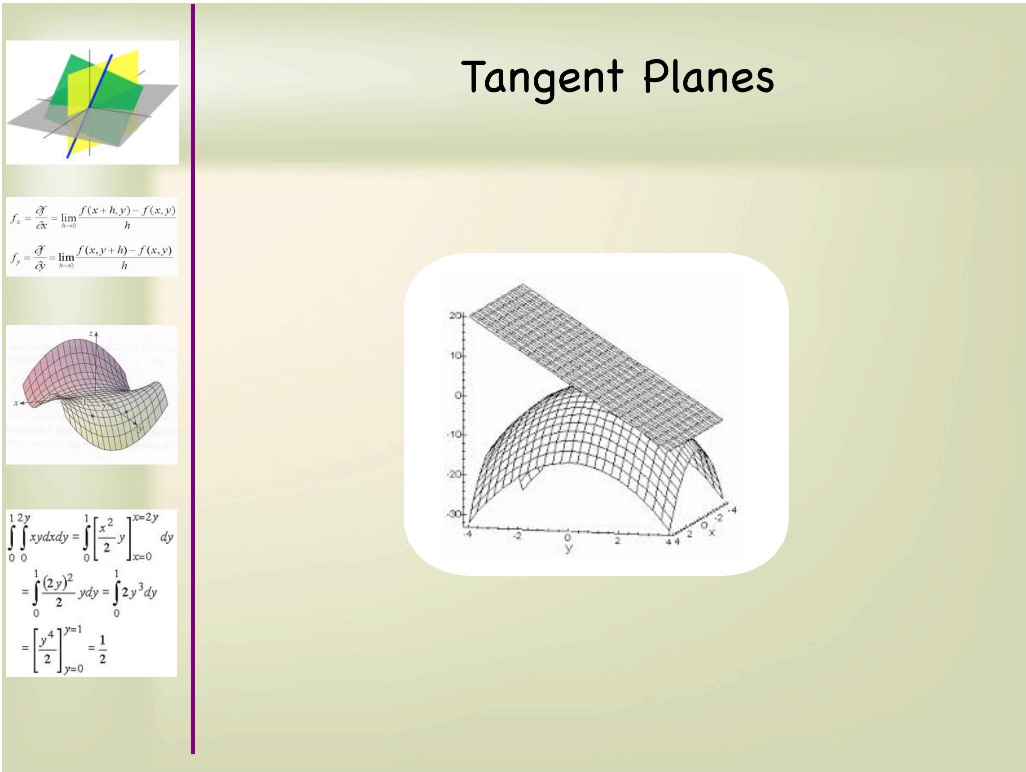

Tangent Planes

We already dealt with tangent planes to surfaces of form $z = f(x,y)$.
Now, we will find tangent planes to surfaces of form $F(x,y,z) = k$,
i.e. a surface represented by any equation in three variables.

Definition

Let $F(x,y,z)=k$ be a surface, F, differentiable at $P(x_0,y_0,z_0)$ with $\nabla F(x_0,y_0,z_0) \neq \vec{0}$. Then the plane through P and perpendicular to $\nabla F(x_0,y_0,z_0)$ is called the tangent plane to the surface at P.

Theorem

For surface $F(x,y,z)=k$, the equation of the tangent plane at (x_0,y_0,z_0) is

$$\nabla F(x,y,z)\cdot\langle x-x_0, y-y_0, z-z_0\rangle = 0$$

$\Leftrightarrow$

$$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0.$$

EX 1 Find the equation of the tangent plane to $8x^2+y^2+8z^2=16$ at $\left(1,2,\frac{\sqrt{2}}{2}\right)$.

EX 2 Find parametric equations of the line that is tangent to the curve of intersection of these surfaces at the point $(1,2,2)$.

$$f(x,y,z) = 9x^2 + 4y^2 + 4z^2 - 41=0$$
$$g(x,y,z) =2x^2 - y^2 + 3z^2 - 10 = 0$$

Definition

Let $z = f(x,y)$, f is differentiable function, dx and dy (differentials) are variables. dz (also called total differential of f) is

$dz = df(x,y) = f_x(x,y)dx + f_y(x,y)dy = \nabla f \cdot \langle dx,dy \rangle.$

EX 3 Use differentials to approximate the change in z as (x,y) moves from P to Q. Also find Δz.

$z = x^2 - 5xy + y \qquad P(2,3) \qquad Q(2.03, 2.98)$

EX 4 Use differentials to find the approximate amount of copper in the four sides and bottom of a rectangular copper tank that is 6 feet long, 4 feet wide and 3 feet deep inside, if the sheet-copper is 1/4 inch thick.

EX 5 A piece of cable (cylindrical) that measures 2 meters long with a radius of 2 centimeters is thought to have measurement error as large as 5 millimeters for each of the height and radius measurements. Estimate the error in the volume measurement.