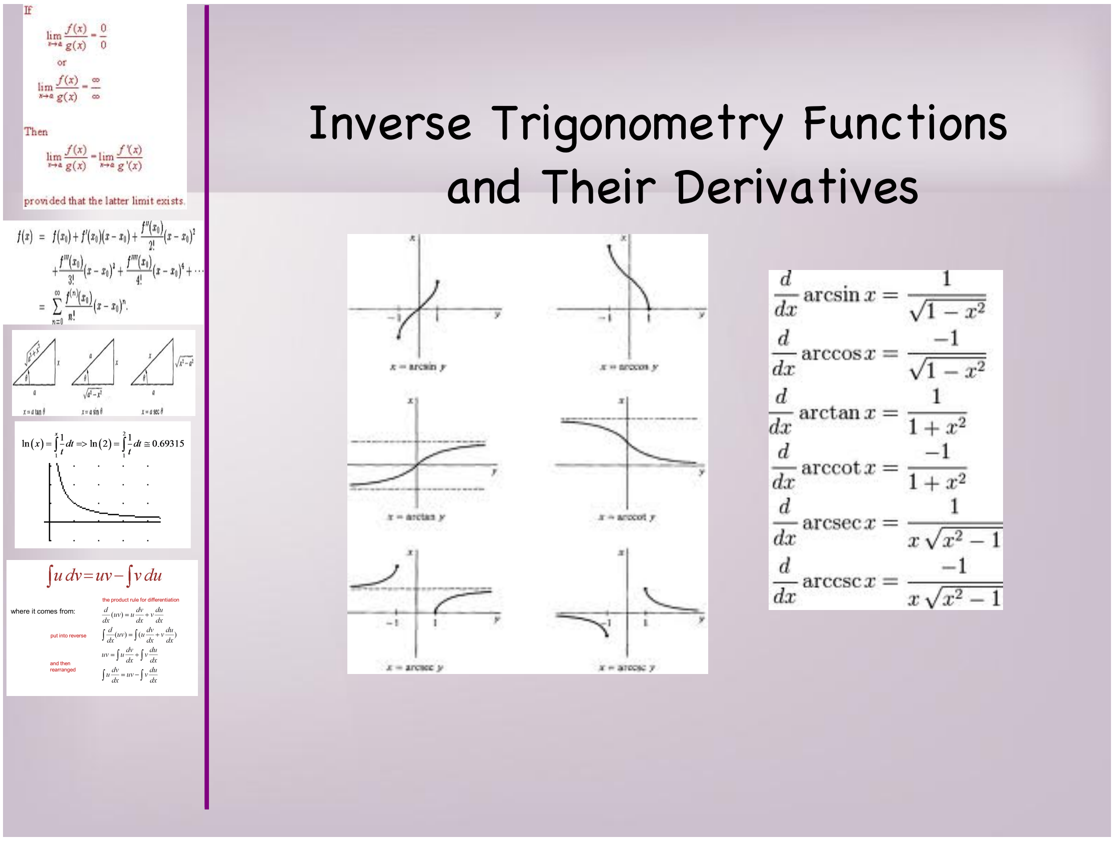
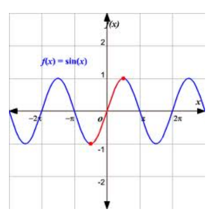
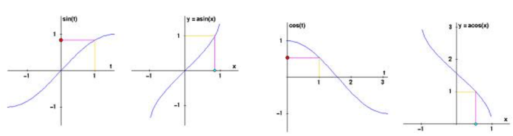



The graph of y = sin x does not pass the horizontal line test, so it has no inverse.




If we restrict the domain (to half a period), then we can talk about an inverse function.



Definition

$$x = \sin^{-1} y \iff y = \sin x \qquad x \epsilon \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$x = \cos^{-1} y \iff y = \cos x \qquad x \epsilon [0, \pi]$$

*notation* $\arccos(x) = \cos^{-1} x$

EX 1 Evaluate these without a calculator.

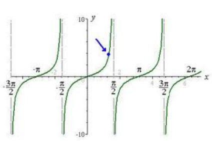
a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

c) $\sin^{-1}(sin(3\pi/2))$

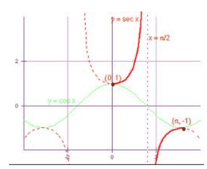
b) $\sin^{-1}(1)$

d) $\cos^{-1}(\cos(-\pi/4))$

*y = tan x*




*y = sec x*

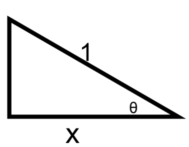



Definition

$$x = \tan^{-1} x \Leftrightarrow y = \tan x \qquad x \in (-\pi/2, \pi/2)$$
$$x = \sec^{-1} y \Leftrightarrow y = \sec x \qquad x \in [0, \pi/2) \cup (\pi/2, \pi]$$

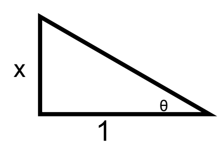
EX 2 Evaluate without a calculator.

a) $\tan^{-1}(-1)$ b) $\sec^{-1}(2)$ c) $\arctan(\tan(\frac{3\pi}{4}))$



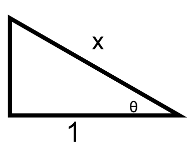

$\theta = \cos^{-1}x$

$\sin(\cos^{-1}x) =$




$\theta = \tan^{-1}x$

$\sec(\tan^{-1}x) =$




$\theta = \sec^{-1}x$

$\tan(\sec^{-1}x) =$

EX 3 Calculate $\sin[2\cos^{-1}(1/4)]$ with no calculator.

$$D_x[\sin x] = \cos x \qquad D_x[\cos x] = -\sin x$$
$$D_x[\tan x] = \sec^2 x \qquad D_x[\cot x] = -\csc^2 x$$
$$D_x[\sec x] = \sec x \tan x \qquad D_x[\csc x] = -\csc x \cot x$$

Derivatives of Inverse Trig Functions

Let $y = \cos^{-1} x$. Find $y'$.

EX 4 $D_x[\tan^{-1}(5x^2 - 3x + 1)] =$

$$D_x[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}},\ x \in (-1,1) \qquad D_x[\cos^{-1} x] = \frac{-1}{\sqrt{1 = x^2}}$$
$$D_x[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}},\ |x| > 1 \qquad D_x[\tan^{-1} x] = \frac{1}{1+x^2}$$

EX 5 Evaluate these integrals.

a) $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

b) $\int \frac{e^x}{1+e^{2x}}\,dx$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$
$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$
$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}|x| + C$$