

If

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$$

or

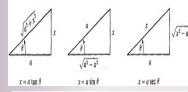
$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$$

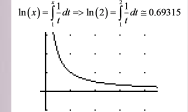
Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

provided that the latter limit exists.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \dots$$

$$= \sum_{k=1}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$


$$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$$


$\int u dv = uv - \int v du$

where it comes from:

The product rule for differentiation

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

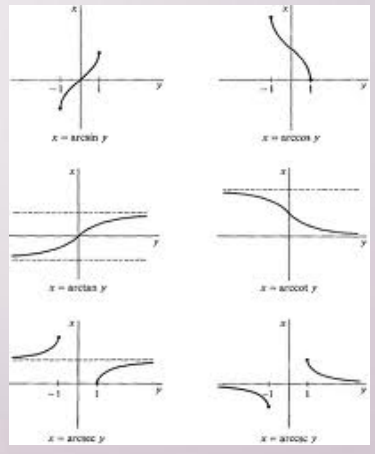
is a nice reverse

$$\int \frac{d}{dx}(uv) = \int \left(u \frac{dv}{dx} + v \frac{du}{dx} \right)$$

and then rearranges

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

Inverse Trigonometry Functions and Their Derivatives



$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$$

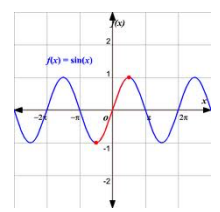
$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \operatorname{arccot} x = \frac{-1}{1+x^2}$$

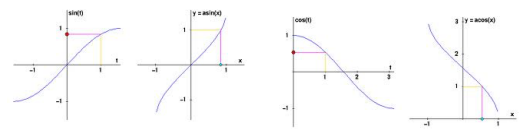
$$\frac{d}{dx} \operatorname{arcsec} x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \operatorname{arccsc} x = \frac{-1}{x\sqrt{x^2-1}}$$

The graph of $y = \sin x$ does not pass the horizontal line test, so it has no inverse.



If we restrict the domain (to half a period), then we can talk about an inverse function.



Definition

$$x = \sin^{-1} y \Leftrightarrow y = \sin x \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$x = \cos^{-1} y \Leftrightarrow y = \cos x \quad x \in [0, \pi]$$

notation $\arccos(x) = \cos^{-1} x$

EX 1 Evaluate these without a calculator.

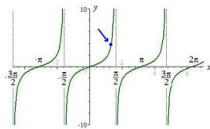
a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

c) $\sin^{-1}(\sin(3\pi/2))$

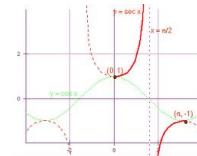
b) $\sin^{-1}(1)$

d) $\cos^{-1}(\cos(-\pi/4))$

$y = \tan x$



$y = \sec x$



Definition

$$x = \tan^{-1} y \Leftrightarrow y = \tan x \quad x \in (-\pi/2, \pi/2)$$

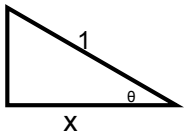
$$x = \sec^{-1} y \Leftrightarrow y = \sec x \quad x \in [0, \pi/2) \cup (\pi/2, \pi]$$

EX 2 Evaluate without a calculator.

a) $\tan^{-1}(-1)$

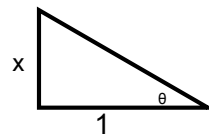
b) $\sec^{-1}(2)$

c) $\arctan(\tan(\frac{3\pi}{4}))$



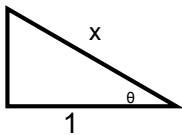
$$\theta = \cos^{-1}x$$

$$\sin(\cos^{-1}x) =$$



$$\theta = \tan^{-1}x$$

$$\sec(\tan^{-1}x) =$$



$$\theta = \sec^{-1}x$$

$$\tan(\sec^{-1}x) =$$

EX 3 Calculate $\sin[2\cos^{-1}(1/4)]$ with no calculator.

Derivatives of Inverse Trig Functions

Let $y = \cos^{-1}x$. Find y' .

$$\begin{aligned}
 D_x[\sin x] &= \cos x & D_x[\cos x] &= -\sin x \\
 D_x[\tan x] &= \sec^2 x & D_x[\cot x] &= -\csc^2 x \\
 D_x[\sec x] &= \sec x \tan x & D_x[\csc x] &= -\csc x \cot x
 \end{aligned}$$

EX 4 $D_x[\tan^{-1}(5x^2 - 3x + 1)] =$

$$\begin{aligned}
 D_x[\sin^{-1} x] &= \frac{1}{\sqrt{1-x^2}}, x \in (-1,1) & D_x[\cos^{-1} x] &= \frac{-1}{\sqrt{1-x^2}} \\
 D_x[\sec^{-1} x] &= \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1 & D_x[\tan^{-1} x] &= \frac{1}{1+x^2}
 \end{aligned}$$

EX 5 Evaluate these integrals.

a) $\int_{-1}^1 \frac{1}{1+x^2} dx$

b) $\int \frac{e^x}{1+e^{2x}} dx$

$$\begin{aligned}
 \int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1} x + C \\
 \int \frac{1}{1+x^2} dx &= \tan^{-1} x + C \\
 \int \frac{1}{x\sqrt{x^2-1}} dx &= \sec^{-1}|x| + C
 \end{aligned}$$