

General Logarithmic and Exponential Functions

1. $\frac{d}{dx} [a^x] = (\ln a)a^x$ 2. $\frac{d}{dx} [a^u] = (\ln a)a^u \frac{du}{dx}$

General Exponential and Logarithmic Functions

$$a^x = \exp(\ln a^x) = \exp(x \ln a) = e^{x \ln a}$$

↔

$$\ln(a^x) = \ln(e^{x \ln a}) = x \ln a$$

Properties of Exponents

(i) $a^x a^y = a^{x+y}$ (iv) $(ab)^x = a^x b^x$

(ii) $\frac{a^x}{a^y} = a^{x-y}$ (v) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

(iii) $(a^x)^y = a^{xy}$

How do we find a derivative or an integral of a^x ?

$$D_x[a^x]=$$

and

$$\int a^x dx =$$

$$D_x[a^x] = a^x \ln a \qquad \int a^x dx = \frac{1}{\ln a}(a^x) + C \qquad a \neq 1$$

$$\begin{array}{l} D_x[a^x] = a^x \ln a \\ \int a^x dx = \frac{1}{\ln a}(a^x) + C \end{array} \quad a \neq 1$$

EX 1 Find y' .

$$y = (2x^3 + 9x)^4 + 4^{2x^3+9x}$$

EX 2 Evaluate $\int \frac{2^{\sqrt{x}}}{3\sqrt{x}} dx$.

Remember log definitions from algebra.

$$y = \log_a x \iff a^y = x$$

Change of base formula

We know $D_x x^a = a x^{a-1}$ is true for rational a . What if a is irrational?

EX 3 Find y' . $y = \sin^2 x + 2^{\sin x}$

EX 4 Find y' . $y = x^x$ Hint: Take the log of both sides.

EX 5 Evaluate $\int_0^1 (10^{3x} + 10^{-3x}) dx$.

EX 6 If $y = (\ln x^2)^{2x+3}$ find y' .