

# Calculus in Polar Coordinates

If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$   
 or  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$   
 provided that the latter limit exists.

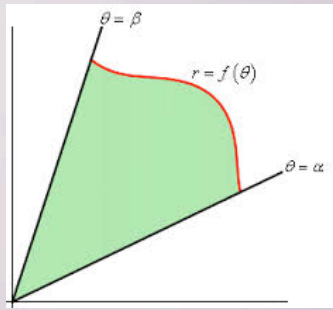
$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$   
 $= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$

$\int u dv = uv - \int v du$

where it comes from:

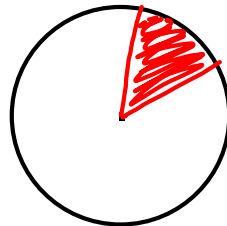
the product rule for differentiation  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$   
 put into reverse  $\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$   
 and then rearrange  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$



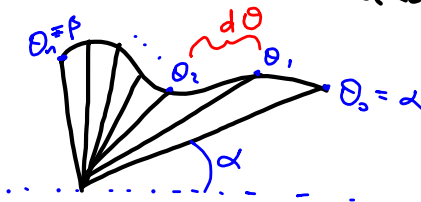
## Calculus in Polar Coordinates

Begin with the area of a sector of a circle:

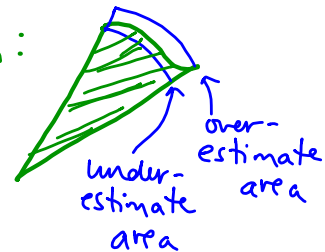
$A =$



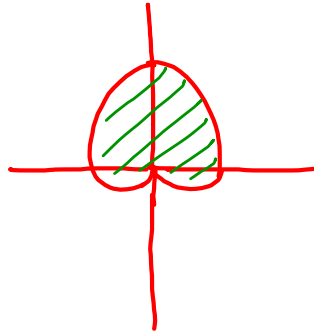
To find area under a curve in the plane



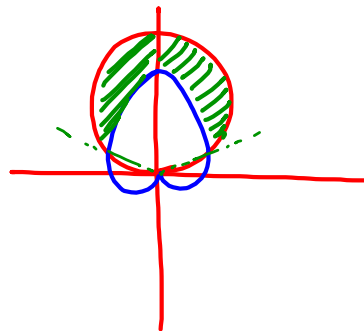
zoom in:



EX 1 Find the area inside  $r = 3 + 3\sin \theta$   
*Cardioid*



EX 2 Find the area inside  $r = 3\sin \theta$  and outside  $r = 1 + \sin \theta$ .  
*circle* *cardioid*



Tangent line slope on a polar curve

$m = \frac{dy}{dx}$  in rectangular coordinates

$$\begin{array}{l} \text{polar coords } \Rightarrow \\ r = f(\theta) \end{array} \left\{ \begin{array}{l} y = r \sin \theta = f(\theta) \sin \theta \\ x = r \cos \theta = f(\theta) \cos \theta \end{array} \right.$$

EX 3 Find the slope of the tangent line to  $r = 2 - 3\sin \theta$  at  $\theta = \pi/6$ .

EX 4 For what values of  $\theta$  will the tangent line to  $r = 2 - 3\sin \theta$  be horizontal?