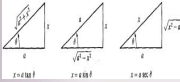


If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$   
 or  
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

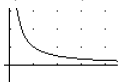
Then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$


$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$



$\int u dv = uv - \int v du$

where it comes from:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

put into reverse:  $\int \frac{d}{dx}(uv) = \int (u \frac{dv}{dx} + v \frac{du}{dx})$

and then rearrange:  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

# Taylor and Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

## Taylor and Maclaurin Series

If we represent some function  $f(x)$  as a power series in  $(x-a)$ , then

### Uniqueness Theorem

Suppose

for every  $x$  in some interval around  $a$ .

Then

### Taylor's Formula with Remainder

Let  $f(x)$  be a function such that  $f^{(n+1)}(x)$  exists for all  $x$  on an open interval containing  $a$ .

Then, for every  $x$  in the interval,

where  $R_n(x)$  is the remainder (or error).

### Taylor's Theorem

Let  $f$  be a function with all derivatives in  $(a-r, a+r)$ .

The Taylor Series

represents  $f(x)$  on  $(a-r, a+r)$

if and only if

EX 1 Find the Maclaurin series for  $f(x)=\cos x$  and prove it represents  $\cos x$  for all  $x$ .

EX 2 Find the Maclaurin series for  $f(x) = \sin x$ .

EX 3 Write the Taylor series for  $f(x) = \frac{1}{x}$  centered at  $a=1$ .

EX 4 Find the Taylor series for  $f(x) = \sin x$  in  $(x-\pi/4)$ .

EX 5 Use what we already know to write a Maclaurin series (5 terms)

for  $e^{-x}$ .