

# Power Series

If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$   
 or  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$   
 Then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$   
 provided that the latter limit exists.

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$   
 $= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$

$\int u dv = uv - \int v du$

where it comes from:  
 $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$   
 put into reverse:  
 $\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + v \frac{du}{dx}$   
 and then rearrange:  
 $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

## Power Series

Consider a series of functions instead of constants.

Power Series in x  $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots \quad a_0 = a_0 x^0$

EX 1 When does this power series converge, i.e., for what x-values?

$$\sum_{n=0}^{\infty} ax^n \quad \left\{ \begin{array}{l} a \in \mathfrak{R} \\ a \neq 0 \end{array} \right\}$$

## **Power Series**

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EX 1 When does this power series converge, i.e., for what x-values?

$$\sum_{n=0}^{\infty} a x^n \quad \begin{cases} a \in \mathfrak{R} \\ a \neq 0 \end{cases}$$

### Theorem

The convergence set for a power series  $\sum_{n=0}^{\infty} a_n x^n$  is always an interval of one of these three types.

- 1) The single point at  $x = 0$ .
- 2) An interval,  $(-R, R)$ ,  $[-R, R]$ ,  $[-R, R)$ , or  $(-R, R]$
- 3)  $(-\infty, \infty)$

The radius of convergence is  $0$ ,  $R$ , or  $\infty$ , respectively.

EX 2 Find the convergence set for  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$

EX 3 Find the convergence set for  $1 + x + \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} + \frac{x^4}{\sqrt{4}} + \dots$  .

Power Series in  $(x-c)$

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \dots$$

Convergence set :

- 1) The single point at  $x = c$
- 2) An interval,  $(c-R, c+R)$  (may include endpoints)
- 3)  $(-\infty, \infty)$

EX 4 Find the convergence set for  $\frac{x-3}{2} + \frac{(x-3)^2}{2^2} + \frac{(x-3)^3}{2^3} + \dots$  .