

# Improper Integrals

if  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{0}{0}$   
or  
 $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$f(x) = f_0(x) + f_1(x)(x-a) + \frac{f_2(x)}{2!}(x-a)^2 + \dots + \frac{f_n(x)}{n!}(x-a)^n + \dots$

$W(x) = \frac{1}{n!} f^{(n)}(a) = W(x) = \frac{1}{n!} f^{(n)}(a) + \dots$

$\int u dv = uv - \int v du$

Example:  $\int_0^{\infty} \frac{1}{1+x^2} dx = \lim_{b \rightarrow \infty} \int_0^b \frac{1}{1+x^2} dx = \lim_{b \rightarrow \infty} [\arctan x]_0^b = \lim_{b \rightarrow \infty} (\arctan b - \arctan 0) = \frac{\pi}{2}$

**Improper Integral** It is like a definite integral except one or both of the limits of integration are  $\pm\infty$ .

**Definition**

$$\int_{-\infty}^b f(x) dx = \lim_{a \rightarrow -\infty} \int_a^b f(x) dx$$

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

**converge** if the limit exists and is finite.  
**diverge** if the limit does not exist (or goes to  $\pm \infty$ ).

EX 1  $\int_{-\infty}^2 e^x dx$

$$\text{EX 2 } \int_1^{\infty} \frac{1}{\sqrt{\pi x}} dx$$

$$\text{EX 3 } \int_1^{\infty} \frac{x}{(1+x^2)^2} dx$$

**Definition** If  $\int_{-\infty}^0 f(x)dx$  and  $\int_0^{\infty} f(x)dx$  converge,

then  $\int_{-\infty}^{\infty} f(x)dx$  converges and

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^0 f(x)dx + \int_0^{\infty} f(x)dx$$

otherwise,  $\int_{-\infty}^{\infty} f(x)dx$  diverges.

$$\text{EX 4 } \int_1^{\infty} \frac{1}{x^p} dx$$

$$\text{EX 5 } \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 16)}$$

$$\text{EX 6 } \int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2 + 16}} dx$$