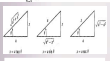


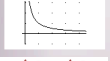
If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$
or
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
provided that the latter limit exists.

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$
 $\frac{f'(a)}{g'(a)} + \frac{f''(a)}{2!g''(a)}(x-a) + \dots$
 $\frac{f''(a)}{2!g''(a)}(x-a)^2 + \dots$



$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$



$\int u dv = uv - \int v du$

Other Indeterminate Forms

Indeterminate forms

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^\infty, \infty^0, 1^\infty$

Other Indeterminate Forms $\frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{x}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 + 4x - 5}{3x^2 - 2x}$$

L'Hopital's rule for $\frac{\infty}{\infty}$

If $\lim_{x \rightarrow a} |f(x)| = \lim_{x \rightarrow a} |g(x)| = \infty$ then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad \text{assuming } \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \text{ exists (finite or } \infty).$$

Now use l'Hopital's Rule for this:

$$\lim_{x \rightarrow \infty} \frac{x}{e^x}$$

$$\text{EX 1 } \lim_{x \rightarrow \infty} \frac{x^9}{e^x}$$

$$\text{EX 2 } \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{2^x}$$

Indeterminate Forms $0 \cdot \infty$ and $\infty - \infty$

$$\text{EX 3 } \lim_{x \rightarrow 0} 3x^2 \csc^2 x$$

$$\text{EX 4 } \lim_{x \rightarrow \pi/2} (\tan x - \sec x)$$

Indeterminate Forms $0^0, \infty^0, 1^\infty$

EX 5 $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

EX 6 $\lim_{x \rightarrow 0^+} x^x$

Indeterminate Forms

NOT Indeterminate Forms

$$\frac{0}{0}$$

$$\frac{\pm\infty}{\pm\infty}$$

$$\frac{0 \cdot \infty}{\infty \cdot \infty}$$

$$\frac{\infty - \infty}{\infty - \infty}$$

$$0^0$$

$$\infty^0$$

$$1^\infty$$

$$1^0$$

$$0^\infty$$

$$\infty^0$$

$$\infty + \infty$$

$$\infty \cdot \infty$$

$$\frac{0}{\infty}$$

$$\frac{\infty}{\infty}$$

$$\frac{\infty}{0}$$