If

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$$

or

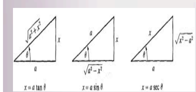
$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\infty}{\infty}$$

Then

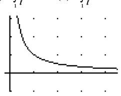
$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

provided that the latter limit exists.

$$\begin{aligned} f(x) &= f(x_0)+f'(x_0)(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2 \\ &\quad +\frac{f'''(x_0)}{3!}(x-x_0)^3+\frac{f''''(x_0)}{4!}(x-x_0)^4+\cdots \\ &= \sum_{n=0}^{\infty}\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n. \end{aligned}$$



$$\ln(x)=\int_1^x \frac{1}{t}\,dt \Rightarrow \ln(2)=\int_1^2\frac{1}{t}\,dt \cong 0.69315$$



$$\int u\,dv = uv-\int v\,du$$

where it comes from: the product rule for differentiation

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

put into reverse

$$\int\frac{d}{dx}(uv)=\int(u\frac{dv}{dx}+v\frac{du}{dx})$$

$$uv=\int u\frac{dv}{dx}+\int v\frac{du}{dx}$$

and then rearranged

$$\int u\frac{dv}{dx}=uv-\int v\frac{du}{dx}$$

Further Practice on Techniques of Integration

$$\int \frac{f(x)}{g(x)}\,dx \qquad \int g(x)f(x)\,dx$$

EX 1 $\displaystyle\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}\,dx$

EX 2 $\int \frac{x^2}{\left(1-9x^2\right)^{3/2}}\,dx$

EX 3 $\int \frac{2x^4-2x^3+6x^2-5x+1}{x^3-x^2+x-1}\,dx$

EX 4 $\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx$

EX 5 $\int \cos(\sqrt{x})\,dx$

EX 6 $\int \frac{\sqrt{4x^2 - 9}}{x} dx$

EX 7 $\int \frac{\sqrt[3]{x+8}}{x} dx$