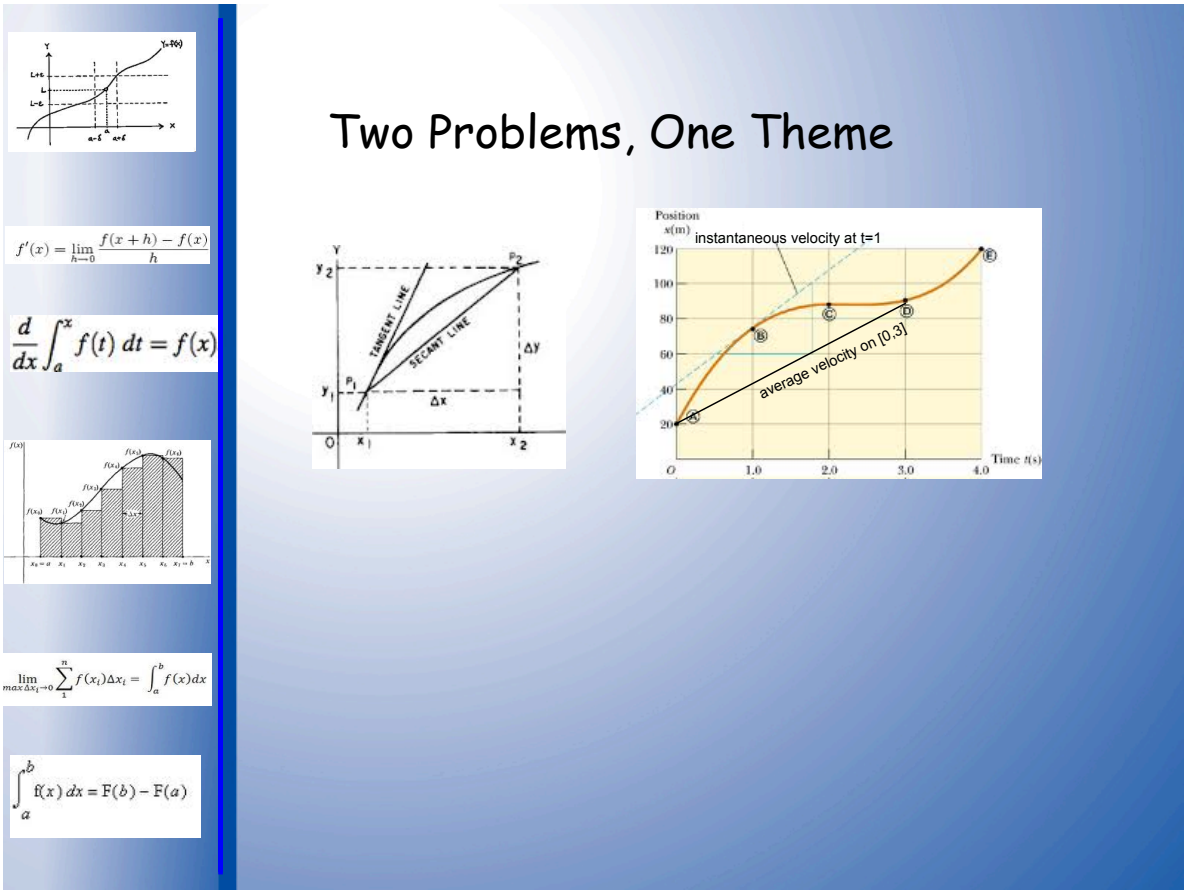
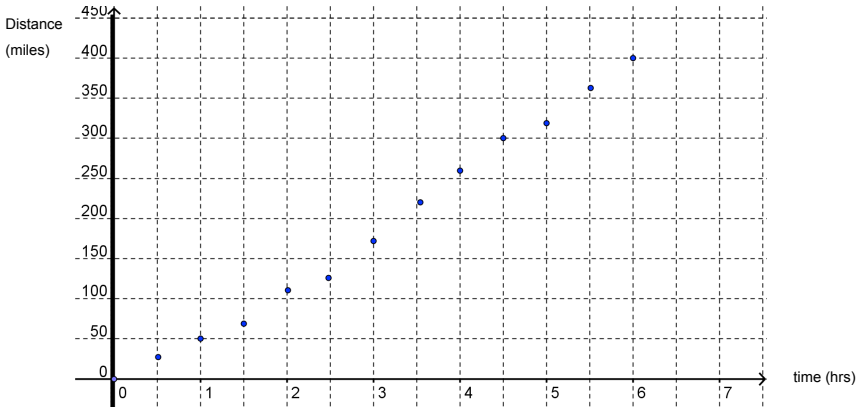
8 Two Problems One Theme



It took me 6 hours to drive 400 miles. As I drove I wrote the mileage on the trip-o-meter each half hour. Here is a graph of my trip.



t	*d*
3	170
2.5	130
2.1	112
2	110

What was my average velocity for the trip? v_{av}=

What was my average velocity for the first half of the trip?

How fast was I going at *t*=2? v_{inst}=

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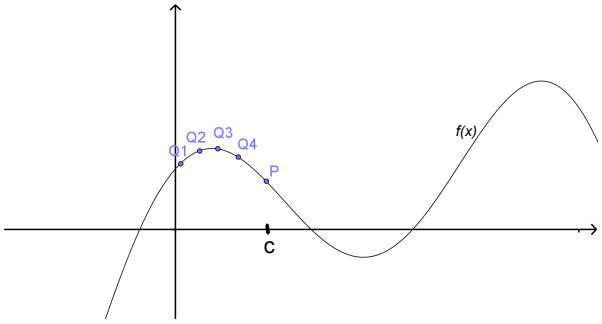
Archimedes - slope of a tangent line.

Kepler, Galileo, Newton - Instantaneous velocity.



Q = "movable" point.

P = Point in question

secant line⇒ line through P and Q.

tangent line⇒ limiting position (if it exists) of secant line as Q moves closer to P along the curve.

slope of secant line

slope of tangent line

EX 1 Find the slope of $y = -x^2 + 3x$ when x = -1, 2, and 5.

EX 2 Find the equation of the tangent line to $y = \frac{2}{x}$ at x=1.

8 Two Problems One Theme

Geometrically finding the slope of a tangent line to a curve is exactly the same as finding the instantaneous velocity for a moving object.

EX 3 An object travels along a line so that its position is given by $s(t)=t^2 + 1$ (measured in meters, t measured in seconds.)

a) What is its average velocity on the interval $2 \le t \le 3$?

b) Average velocity on $2 \le t \le 2.003$?

c) Average velocity on $2 \le t \le 2+h$?

d) Instantaneous velocity at $t=2$?

"Rate of change" means instantaneous rate of change.