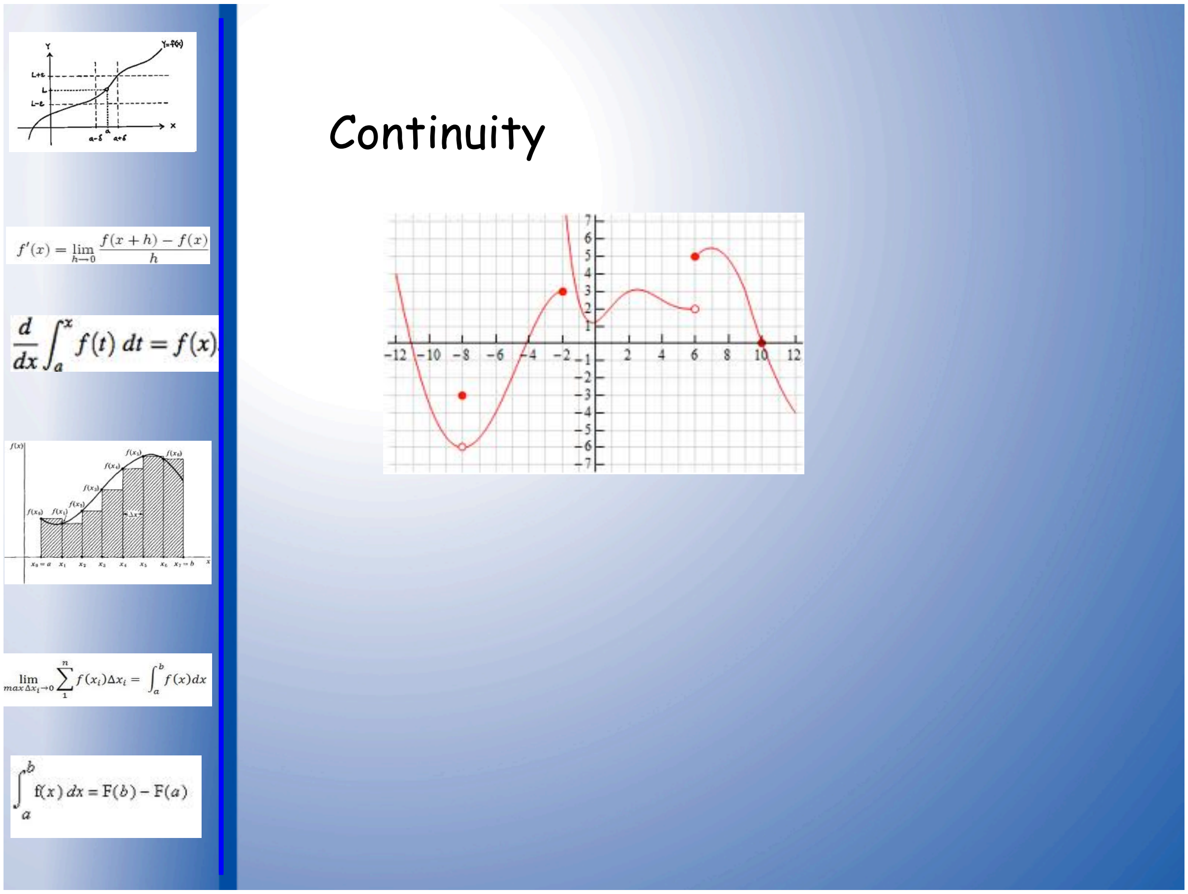
6 Continuity



Definition: Continuity at a Point

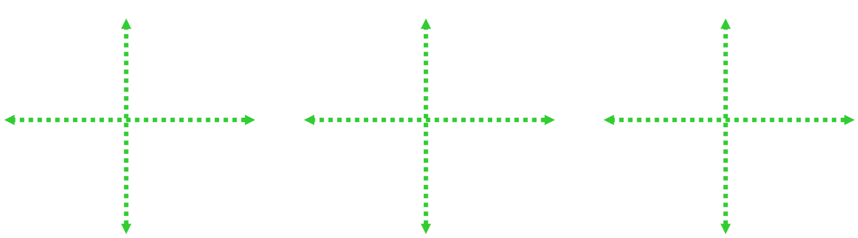
Let *f* be defined on an open interval containing *c*. We say that *f* is continuous at *c* if

$$\lim_{x\to c} f\left(x\right) = f(c)$$

This indicates three things:

1. The function is defined at $x = c$.
2. The limit exists at $x = c$.
3. The limit at $x = c$ needs to be exactly the value of the function at x = c.

Three examples:



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Continuous Functions

a) All polynomial functions are continuous everywhere.

b) All rational functions are continuous over their domain.

c) The absolute value function is continuous everywhere.

d) $f(x)=\sqrt[n]{x}$ is continuous for all real numbers if n is odd.

e) $f(x)=\sqrt[n]{x}$ is continuous for all non-negative real numbers if n is even.

f) The sine and cosine functions are continuous over all real numbers.

g) The cotangent, cosecant, secant and tangent functions are continuous over their domain.

More continuous functions

If *f(x)* and *g(x)* are continuous at *x* = *c*, then so are

$$kf(x),\ (f \pm g)(x),\ (fg)(x),\quad \frac{f}{g}(x),\ (g(x) \neq 0),$$

$$f^n(x),\quad \sqrt[n]{f(x)}\ ,\left(f(c) > 0\ if\ n\ is\ even\right).$$

EX 1 State where these functions are continuous.

a) $f(x)=x^2-9$

b) $g(x)=\sqrt{x-5}$

c) $h(x)=\dfrac{21-7x}{x-3}$

d) $p(x)=\begin{cases} 7-3x & x \leq 3 \\ -2 & x > 3 \end{cases}$

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Composite Limit Theorem

If $\lim_{x \to c} g(x) = L$ and *f* is continuous at *L*, then

$$\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x)) = f(L)$$

Ex 2 At what points are the following functions continuous?

a) $h(x) = \frac{1}{\sqrt{4+x^2}}$

b) $g(t) = |t-2|$

Ex 3 If $f(x) = \frac{x^2 - 49}{x-7}$, how do we need to complete the definition for this to be continuous everywhere?

Intermediate Value Theorem

f is a function defined on *[a,b]* and *ω* is a number between *f(a)* and *f(b)*.

If *f* is continuous on *[a,b]*, then there exists at least one number, *c, (a<c<b)* such that *f(c)* = *ω*.

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Use interval notation to state all values for which this function is continuous.