

## Limits At Infinity, Infinite Limits

Definition: (Limit as $x \rightarrow \infty$ )
is defined on $[\mathrm{c}, \infty)$ for $\mathrm{c} \in \mathrm{R}$
We say that if for every $\varepsilon>0$ there is a corresponding number, $m$ such that


EX 1 Intuitively (looking at the graph) determine these limits.

$$
\begin{aligned}
& \lim _{x \rightarrow \infty} f(x)= \\
& \lim _{x \rightarrow-\infty} f(x)=
\end{aligned}
$$



# EX 2 Show that if $n$ is a positive integer, then $\lim _{x \rightarrow \infty} \frac{1}{x^{n}}=0$ 

EX $3 \lim _{x \rightarrow \infty} \frac{2 x+3}{x^{2}+1}=$

EX $4 \quad \lim _{x \rightarrow \infty} \frac{3 x^{4}-2 x^{3}+53}{x^{3}+7}=$
\#X $5 \quad \lim _{x \rightarrow \infty} \frac{2 x^{2}+5 x-1}{x^{2}+3 x}=$

## Definition: (Infinite limit)

We say $\quad \lim _{x \rightarrow c^{+}} f(x)=\infty \quad$ if for every positive number, m
there is a corresponding $\delta>0$ such that $0<x-c<\delta \Rightarrow f(x)>m$

EX 6 Determine these limits looking at this graph of $f(x)=\frac{1}{x-1}$.


$$
\begin{array}{ll}
\lim _{x \rightarrow \infty} f(x)= & \lim _{x \rightarrow 1^{+}} f(x)= \\
\lim _{x \rightarrow-\infty} f(x)= & \lim _{x \rightarrow 1^{-}} f(x)=
\end{array}
$$

Ex 7 Find the horizontal and vertical asymptotes for this function, then write a few limit statements including $\infty . \quad f(x)=\frac{-2 x}{x+3}$

Ex 8 a) Find the vertical and horizontal asymptotoes for this function.

$$
f(x)=\frac{2 x}{\sqrt{x^{2}+5}}
$$

b) Determine these limits:

$$
\begin{array}{ll}
\lim _{x \rightarrow \infty} f(x)= & \lim _{x \rightarrow \sqrt{5}^{+}} f(x)= \\
\lim _{x \rightarrow-\infty} f(x)= & \lim _{x \rightarrow \sqrt{5}} f(x)=
\end{array}
$$

Determine these limits:

$\lim _{x \rightarrow \infty} f(x)=$
$\lim _{x \rightarrow 0^{+}} f(x)=$
$\lim _{x \rightarrow-\infty} f(x)=$
$\lim _{x \rightarrow 0^{-}} f(x)=$

