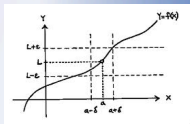
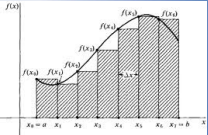


30 Volume Solid



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

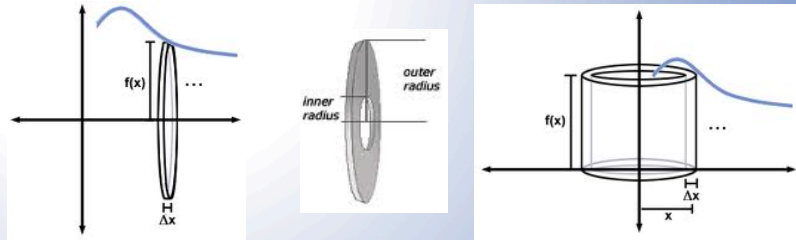
$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



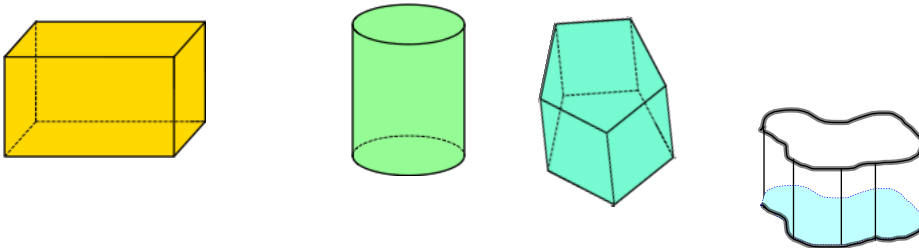
$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Volume of Solids



The volume of a solid right prism or cylinder is the area of the base times the height.



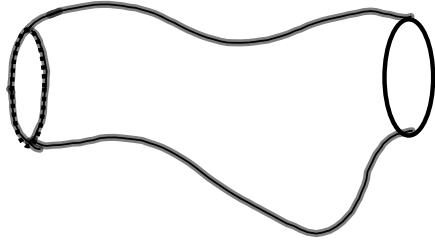
Volume of one penny:

Volume of a stack of pennies:

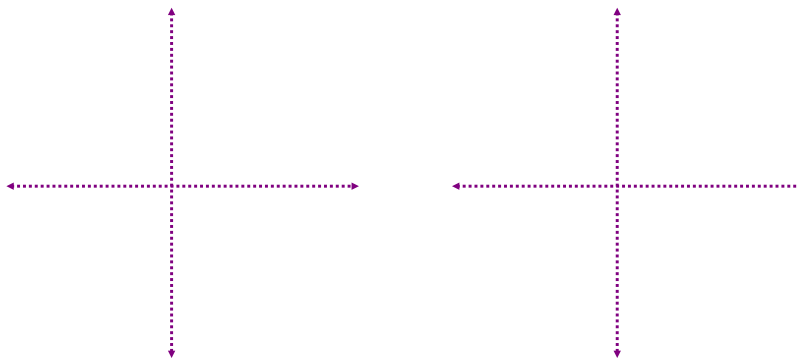
30 Volume Solid

Disk Method

How would we find the volume of a shape like this?

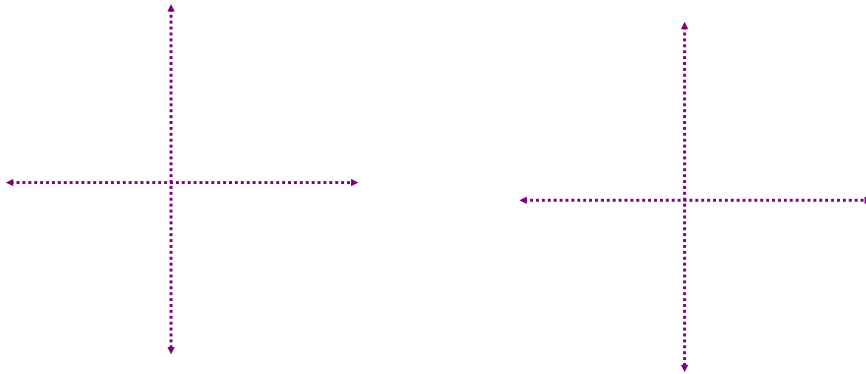


EX 1 Find the volume of the solid of revolution obtained by revolving the region bounded by $y = \sqrt{x}$, the x -axis and the line $x=9$ about the x -axis.



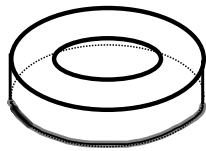
30 Volume Solid

EX 2 Find the volume of the solid generated by revolving the region enclosed by $x = \frac{2}{y}$, $y = 2$, $y = 6$, and $x = 0$ about the y -axis.



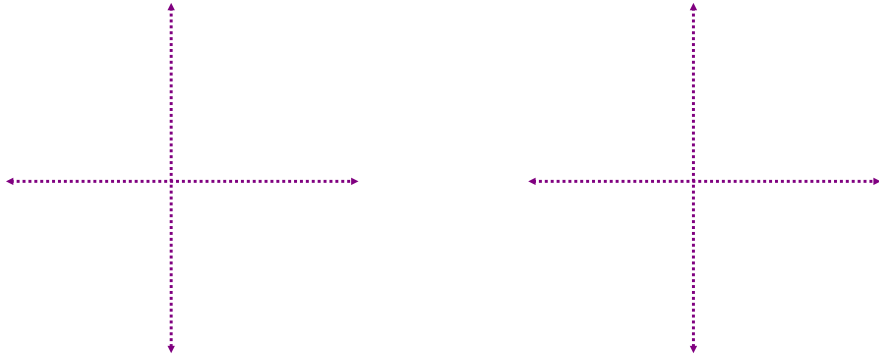
Washer Method

How would we find the volume of a washer?



30 Volume Solid

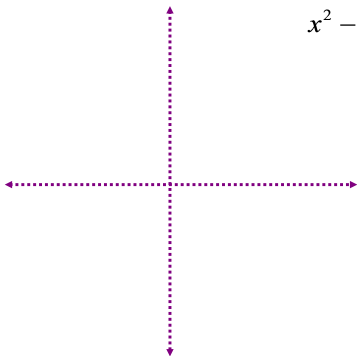
EX 3 Find the volume of the solid generated by revolving about the x-axis the region bounded by $y = 6x$ and $y = 6x^2$.



EX 4 Find the volume of the solid generated by revolving about the line $y = 2$ the region in the first quadrant bounded by these parabolas and the y-axis. (Hint: Always measure radius from the axis of revolution.)

$$3x^2 - 16y + 48 = 0$$

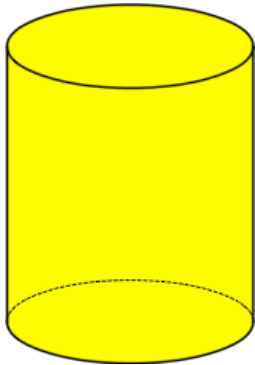
$$x^2 - 16y + 80 = 0$$



30 Volume Solid

Shell Method

How would we find the volume of a label we peel off a can?

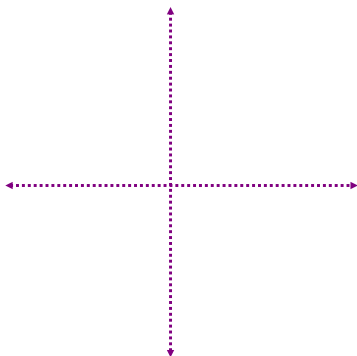


EX 5 Find the volume of the solid generated when the region bounded by these three equations is revolved about the y -axis.

$$y = x^2$$

$$x = 1$$

$$y = 0$$

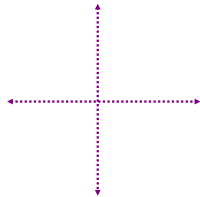


30 Volume Solid

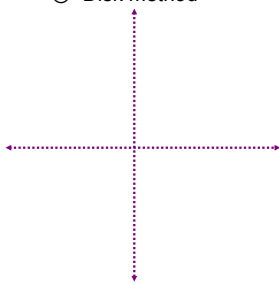
EX 6 Find the volume of the solid generated when the region in the first quadrant bounded by these equations is revolved about the y -axis in two ways.

$$y = 9 - x^2, x \geq 0 \quad x = 0 \quad y = 0$$

① Shell method

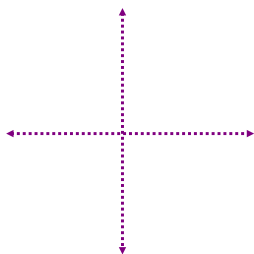


② Disk method



EX 7 Find the volume of the solid generated when the region in the first quadrant bounded by these equations is revolved about the line $x = 3$.

$$y = 9 - x^2, x \geq 0 \quad x = 0 \quad y = 0$$



30 Volume Solid

EX 8 A region, R is shown below. Set up an integral for the volume obtained by revolving R about the given line.

- The y -axis
- The x -axis
- The line $y = 3$

