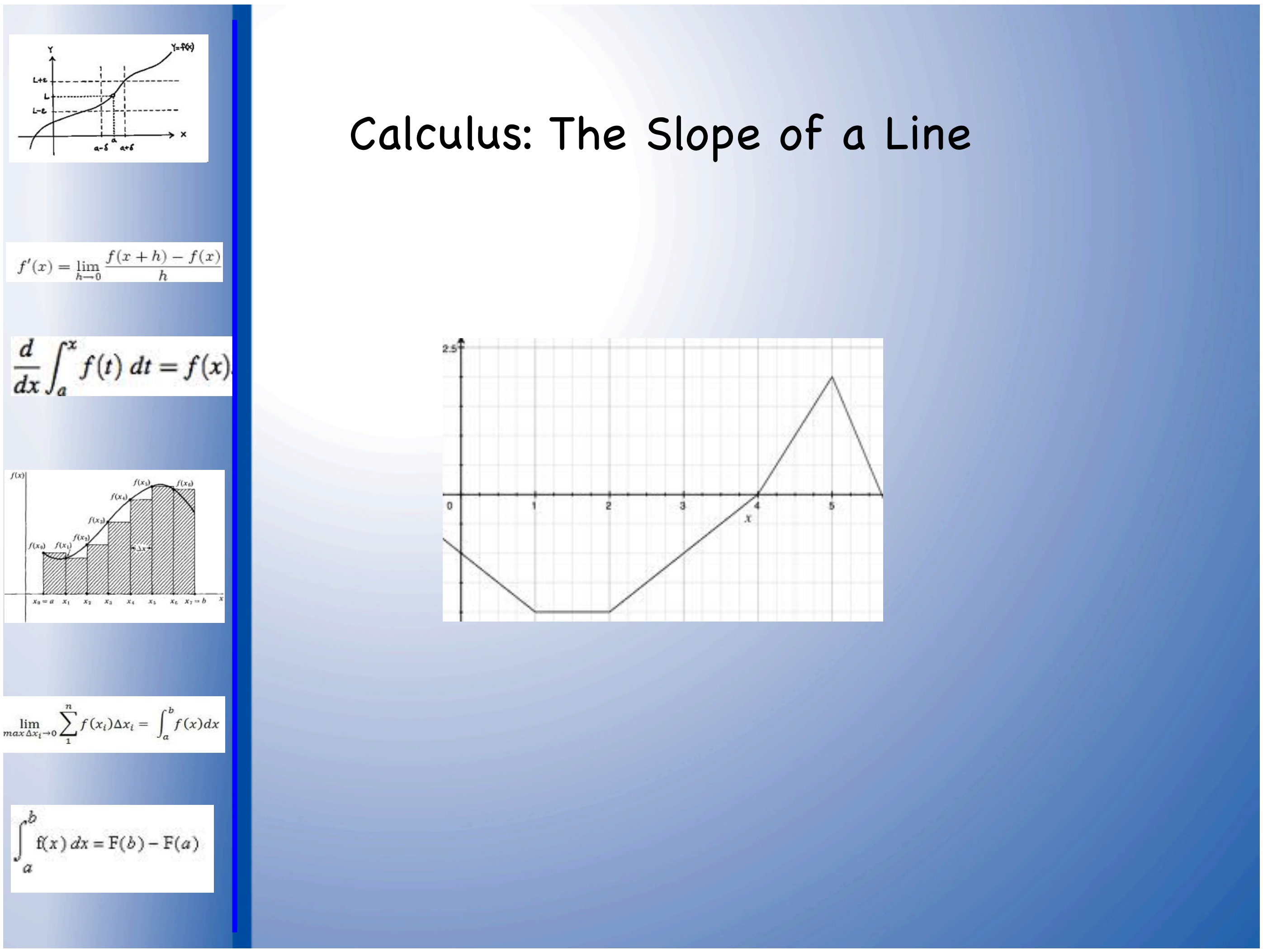

Calculus: The Slope of a Line
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{max\, \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$

1B Slope of a Line

There is only one line between any 2 points.

The slope of a line is:

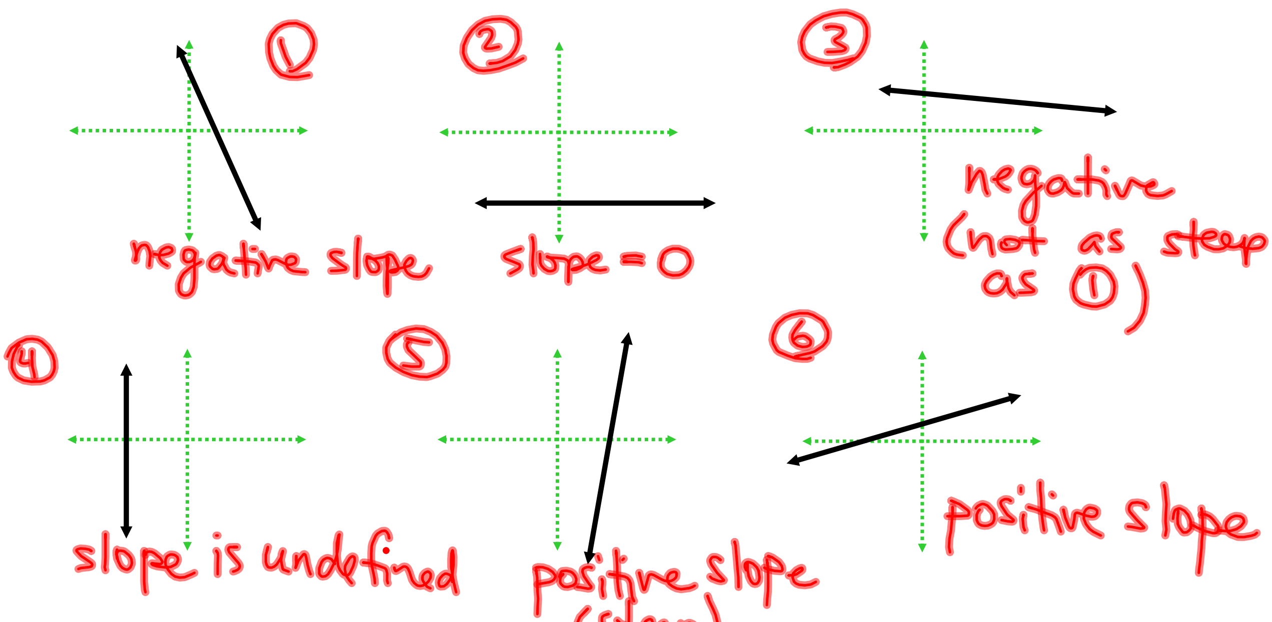
The steepness of the line. (same steepness everywhere on the line)

The vertical change over the horizontal change, denoted by *m*.

Given two points, (x_1,y_1), (x_2,y_2) in the Cartesian Plane,

$$\boxed{m = \frac{y_2-y_1}{x_2-x_1}}$$

Examples of slope:



EX 1

a) Find the slope of the line containing these points: $\overset{x_1\,y_1}{(-3,2)}$ and $\overset{x_2\,y_2}{(2,5)}$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{y_1-y_2}{x_1-x_2} = \frac{5-2}{2-(-3)} = \boxed{\frac{3}{5}}$$

b) Find the slope of the line containing these points: $\underset{x_1\,y_1}{(5,-6)}$ and $\underset{x\,y_2}{(-2,-6)}$

$$m = \frac{-6-(-6)}{-2-5} = \frac{0}{-7} = \boxed{0}$$

Point-Slope Form of a Line (Eqns of lines)

Given that m = the slope of a line and it goes through the point (x_1, y_1), then we know:

$$\boxed{y-y_1=m(x-x_1)}$$

comes from

$$m=\frac{y-y_1}{x-x_1}$$

$$(x-x_1)m=y-y_1$$

Slope-Intercept Form of a Line

Given that the slope of a line is m and the y-intercept is the point $(0,b)$, then the equation of the line is:

$$\boxed{y=mx+b}$$

$$m=\frac{y-b}{x-0}$$

$$m=\frac{y-b}{x}$$

$$mx=y-b \Leftrightarrow y=mx+b$$

EX 2

a) Find the equation of the line going through $(-4,1)$ and $(5,2)$.

$x_1\ y_1 \quad x_2\ y_2$

$$m=\frac{2-1}{5-(-4)}=\frac{1}{9}$$

pt-slope:

$$y-1=\frac{1}{9}(x-{}^{-}4)$$

$$y-1=\frac{1}{9}x+\frac{4}{9} \Rightarrow \boxed{y=\frac{1}{9}x+\frac{13}{9}}$$

b) Find the equation of the line with slope, $m = 3$ and y-intercept $(0,5)$.

slope-intercept form:

$$\boxed{y=3x+5}$$

General Equation of a Line

Every line can be written in the form $Ax + By + C = 0$, where $A, B,$ *and* C are integers.

(I prefer slope-intercept form.)

EX 3

Write the equations from Exercise 2 in general form.

1. $y = \frac{1}{9}x + \frac{13}{9}$

$9y = 9\left(\frac{1}{9}x + \frac{13}{9}\right)$

$9y = x + 13$

$-x + 9y - 13 = 0$

or $x - 9y + 13 = 0$

2. $y = 3x + 5$

$-3x + y - 5 = 0$

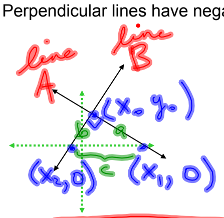
or $3x - y + 5 = 0$

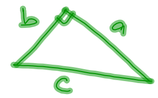
1B Slope of a Line

Parallel and Perpendicular Lines

Parallel lines have the same slope.

Perpendicular lines have negative reciprocal slopes.





use Pythagorean Thm

$$a^2+b^2=c^2$$

$$\boxed{a^2=(x_0-x_1)^2+(y_0-0)^2=(x_0-x_1)^2+y_0^2}$$

length squared from $(x_1,0)$ to (x_0,y_0)

$$\boxed{b^2=(x_2-x_0)^2+(0-y_0)^2=(x_2-x_0)^2+y_0^2}$$

$(x_2,0)$ to (x_0,y_0)

$$\boxed{c^2=(x_2-x_1)^2+(0-0)^2=(x_2-x_1)^2}$$

$(x_2,0)$ to $(x_1,0)$

Plug into Pyth. Thm.

$$(x_0-x_1)^2+y_0^2+(x_2-x_0)^2+y_0^2=(x_2-x_1)^2$$

$$x_0^2-2x_0x_1+\cancel{x_1^2}+y_0^2+\cancel{x_2^2}-2x_0x_2+x_0^2+y_0^2=\cancel{x_2^2}-2x_1x_2+\cancel{x_1^2}$$

$$\tfrac{1}{2}\left(2y_0^2+2x_0^2-2x_0x_1-2x_0x_2\right)=\left(-2x_1x_2\right)\tfrac{1}{2}$$

$$y_0^2+x_0^2-x_0x_1-x_0x_2=-x_1x_2$$

$$y_0^2=\underbrace{-x_0^2+x_0x_1}+\underbrace{x_0x_2-x_1x_2}$$

$$y_0^2=-x_0\underline{(x_0-x_1)}+x_2\underline{(x_0-x_1)}$$

$$\frac{y_0^2}{y_0(x_2-x_0)}=\frac{(x_0-x_1)\cancel{(x_2-x_0)}}{y_0\cancel{(x_2-x_0)}}$$

$$\frac{y_0}{x_2-x_0}=\frac{x_0-x_1}{y_0}$$

slope of line A: $m_A=\dfrac{y_0-0}{x_0-x_1}=\dfrac{y_0}{x_0-x_1}$

(x_0,y_0) $(x_1,0)$

slope of line B: $m_B=\dfrac{y_0-0}{x_0-x_2}=\dfrac{y_0}{x_0-x_2}$

(x_0,y_0) $(x_2,0)$

$$\Rightarrow \frac{y_0}{x_2-x_0}=-\left(\frac{y_0}{x_0-x_2}\right)=-m_B$$

$$\Rightarrow \frac{x_0-x_1}{y_0}=\frac{1}{m_A}$$

$$\Rightarrow -m_B=\frac{1}{m_A} \Leftrightarrow m_B=\frac{-1}{m_A}$$

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EX4

a) Find the equation of the line parallel to *3x - 4y = 8* which passes through the point *(1,3)*.

$x_1\ y_1$

$m = \frac{3}{4}$

$y - 3 = \frac{3}{4}(x-1)$

$y = \frac{3}{4}x - \frac{3}{4} + 3$

$y = \frac{3}{4}x + \frac{9}{4}$

$\hookrightarrow -4y = -3x + 8 \qquad / \div(-4)$

$y = \frac{3}{4}x - 2$

$\Rightarrow$ slope $= \frac{3}{4}$

b) Find the equation of the line perpendicular to *y = -3x + 5* which passes through the origin.

thru (0,0) (y-intercept)

$m = \frac{1}{3}$

$y = \frac{1}{3}x + 0$

$\Leftrightarrow y = \frac{1}{3}x$

slope $= -3$

$\perp$ slope $= -\left(\frac{1}{-3}\right) = \frac{1}{3}$

1B Slope of a Line

Determine the slope of each line segment in this function.