

## Implicit Derivatives

$$
\begin{aligned}
& \frac{x^{2}}{9}+\frac{y^{2}}{4}=2 \\
& \frac{1}{9} \frac{d}{d x} x^{2}+\frac{1}{4} \frac{d}{d x} y^{2}=\frac{d}{d x} 2 \\
& \frac{1}{9} 2 x+\frac{1}{4} \frac{d y}{d x} 2 y=0 \\
& \frac{2 y}{4} \frac{d y}{d x}=-\frac{2 x}{9} \\
& \frac{d y}{d x}=\frac{-2 x(4)}{9(2 y)} \\
& \frac{d y}{d x}=\frac{-8 x}{18 y}=\frac{-4 x}{9 y}
\end{aligned}
$$

Given the equation $2 y^{3}-y^{2}=x^{2}+5$
How do we find $\frac{d y}{d x}$ ?

## Let's check to see if implicit differentiation is reasonable.

Differentiate $x^{2}+2 x^{2} y+3 x y=0$ in two ways.

Implicit

Explicit

EX 1 Find $\frac{d y}{d x}$ for the following equations.
a) $x \sqrt{y+1}=x y+1$
b) $9 x^{2}+4 y^{2}=36$

To convince ourselves that is works, let's look at a familiar equation.
Find the equation of the tangent line at the point $(-4,3)$ on this circle.

$$
x^{2}+y^{2}=25 \text { - }
$$

$$
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$$

EX 2 Find the equation of the tangent line at the indicated point.

$$
y+\cos \left(x y^{2}\right)+3 x^{2}=4 \quad \text { at } \quad(1,0)
$$

Power Rule (revisited): Basically the power rule can now be used with rational exponents.

EX $3 \quad$ Find $y^{\prime}$ if $\quad y=\sqrt[3]{x}-2 x^{\frac{7}{2}}$

$$
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& \frac{1}{9} 2 x+\frac{1}{4} \frac{d y}{d x} 2 y=0 \\
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