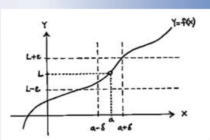
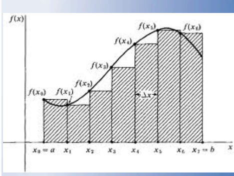


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{max\,\Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Implicit Derivatives

$$\frac{x^2}{9} + \frac{y^2}{4} = 2$$

$$\frac{1}{9}\frac{d}{dx}x^2 + \frac{1}{4}\frac{d}{dx}y^2 = \frac{d}{dx}2$$

$$\frac{1}{9}2x + \frac{1}{4}\frac{dy}{dx}2y = 0$$

$$\frac{2y}{4}\frac{dy}{dx} = -\frac{2x}{9}$$

$$\frac{dy}{dx} = \frac{-2x(4)}{9(2y)}$$

$$\frac{dy}{dx} = \frac{-8x}{18y} = \frac{-4x}{9y}$$

Given the equation $2y^3 - y^2 = x^2 + 5$

How do we find $\frac{dy}{dx}$? $\begin{pmatrix} x \text{ input} \\ y \text{ output} \end{pmatrix}$

y is given as a fn of x, implicitly (relation)
(we cannot solve for y by itself)

($\frac{dy}{dx}$ derivative of dep. variable w.r.t. indep variable)

so far, we've seen fns like $y = f(x)$
(y as an <u>explicit</u> fn of x)

$(y = y(x))$

$$2y^3 - y^2 = x^2 + 5$$

$$D_x(2y^3 - y^2) = D_x(x^2 + 5)$$

$$6y^2\left(\frac{dy}{dx}\right) - 2y\left(\frac{dy}{dx}\right) = 2x$$

$$\frac{dy}{dx}\left(6y^2 - 2y\right) = 2x$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{6y^2 - 2y}}$$

<u>note</u>: derivative formula now depends on x <u>and</u> y.
(for explicit fns we get y' as a fn of only x)

note of warning:
implicit "fns" are not always fns; sometimes they're just relations

Let's check to see if implicit differentiation is reasonable.

Differentiate $x^2 + 2x^2 y + 3xy = 0$ in two ways.

Implicit

$$D_x\left(x^2 + 2x^2 y + 3xy\right) = D_x(0)$$

$$2x + \left(4xy + 2x^2 \frac{dy}{dx}\right) + \left(3y + 3x\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(2x^2+3x\right) = -2x-4xy-3y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-4xy-3y}{2x^2+3x}}$$

plug y into here

Explicit

$$x^2 + 2x^2 y + 3xy = 0$$
$$-x^2 \qquad\qquad -x^2$$

$$y\left(2x^2+3x\right) = -x^2$$

$$y = \frac{-x^2}{2x^2+3x}$$

$$D_x(y) = D_x\left(\frac{-x^2}{2x^2+3x}\right)$$

$$y' = \frac{(2x^2+3x)(-2x) - (-x^2)(4x+3)}{(2x^2+3x)^2}$$

EX 1 Find $\frac{dy}{dx}$ for the following equations.

$\sqrt{y+1} = (y+1)^{1/2}$

a) $x\sqrt{y+1} = xy + 1$

$$D_x\left(x\sqrt{y+1}\right) = D_x\left(xy+1\right)$$

$$1\cdot\sqrt{y+1} + x\left(\frac{1}{2}(y+1)^{-1/2}\left(\frac{dy}{dx}\right)\right) = 1\cdot y + x\frac{dy}{dx}$$

$$\sqrt{y+1} + \frac{x}{2\sqrt{y+1}}\frac{dy}{dx} - x\frac{dy}{dx} = y$$

$$\boxed{\frac{dy}{dx} = \frac{y-\sqrt{y+1}}{\frac{x}{2\sqrt{y+1}} - x}}$$

b) $9x^2+4y^2 = 36$

$$D_x\left(9x^2+4y^2\right) = D_x(36)$$

$$18x + 8y\frac{dy}{dx} = 0$$

$$8y\frac{dy}{dx} = -18x$$

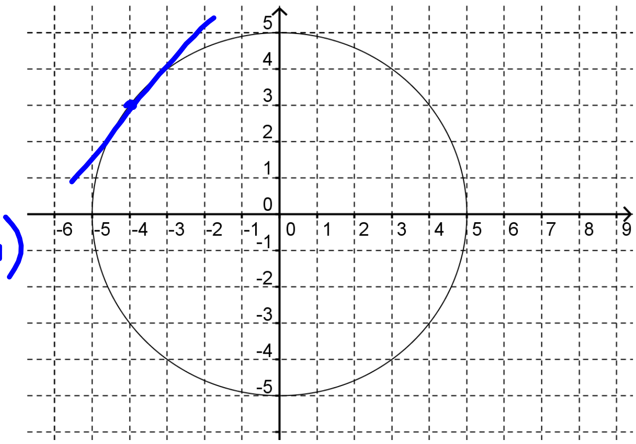
$$\frac{dy}{dx} = \frac{-18x}{8y}$$

$$\boxed{\frac{dy}{dx} = \frac{-9x}{4y}}$$

To convince ourselves that is works, let's look at a familiar equation.

Find the equation of the tangent line at the point (-4,3) on this circle.

$x^2+y^2=25$



① need pt (-4,3)

② need slope

③ plug into $y-y_1=m(x-x_1)$

② $D_x(x^2+y^2)=D_x(25)$

$$2x+2y\left(\frac{dy}{dx}\right)=0$$

$$2y\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

(pt) (-4,3)

$$m=\frac{-x}{y}\Big|_{(-4,3)}=\frac{-(-4)}{3}=\frac{4}{3}$$

$x^2+y^2=25$

③ $y-3=\frac{4}{3}(x-(-4))$

$$y-3=\frac{4}{3}x+\frac{16}{3}$$

$$\boxed{y=\frac{4}{3}x+\frac{25}{3}}$$

$$3=\frac{9}{3}$$

EX 2 Find the equation of the tangent line at the indicated point.

$y+\cos\left(xy^2\right)+3x^2=4$ at (1,0)

① need slope
② eqn line

① $D_x\left(y+\cos\left(xy^2\right)+3x^2\right)=D_x(4)$

$\frac{dy}{dx}+-\sin(xy^2)\left(1\cdot y^2+x(2y)\left(\frac{dy}{dx}\right)\right)+6x=0$

$\frac{dy}{dx}-y^2\sin(xy^2)-2xy\sin(xy^2)\frac{dy}{dx}=-6x$

$\frac{dy}{dx}\left(1-2xy\sin(xy^2)\right)=y^2\sin(xy^2)-6x$

$$\frac{dy}{dx}=\frac{y^2\sin(xy^2)-6x}{1-2xy\sin(xy^2)}$$

at (1,0)

$m=\frac{0^2\sin(0)-6(1)}{1-2(1)(0)}=\frac{-6}{1}=-6$

②

eqn: $y-0=-6(x-1)$

$\boxed{y=-6x+6}$

Power Rule (revisited): Basically the power rule can now be used with rational exponents.

Note: so far, we only showed $D_x(x^n) = nx^{n-1}$ for $n \in \mathbb{Z}$.

$y = x^{m/n}$ $\quad m, n \in \mathbb{Z},\ n \neq 0$

$y^n = x^m$

$D_x(y^n) = D_x(x^m)$

$ny^{n-1}\frac{dy}{dx} = mx^{m-1}$

$\frac{dy}{dx} = \frac{mx^{m-1}}{ny^{n-1}}$ $\quad$ (but $y = x^{m/n}$)

$= \frac{m x^{m-1}}{n(x^{m/n})^{n-1}} = \frac{m x^{m-1}}{n x^{m/n(n-1)}}$

$= \frac{m}{n} x^{m-1-\frac{m}{n}(n-1)}$

$= \frac{m}{n} x^{\cancel{m}-1-\cancel{m}+\frac{m}{n}}$

$y' = \frac{m}{n} x^{\frac{m}{n}-1}$

$$\boxed{D_x(x^{m/n}) = \frac{m}{n} x^{\frac{m}{n}-1}}$$

EX 3 Find y' if $y = \sqrt[3]{x} - 2x^{\frac{7}{2}}$ $= x^{1/3} - 2x^{7/2}$

$y' = \frac{1}{3} x^{\frac{1}{3}-1} - 2\left(\frac{7}{2}\right) x^{\frac{7}{2}-1}$

$y' = \frac{1}{3} x^{-2/3} - 7x^{5/2}$

$$\boxed{\frac{x^2}{9}+\frac{y^2}{4}=2}$$

$$\frac{1}{9}\frac{d}{dx}x^2+\frac{1}{4}\frac{d}{dx}y^2=\frac{d}{dx}2$$

$$\frac{1}{9}2x+\frac{1}{4}\frac{dy}{dx}2y=0$$

$$\frac{2y}{4}\frac{dy}{dx}=-\frac{2x}{9}$$

$$\frac{dy}{dx}=\frac{-2x(4)}{9(2y)}$$

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