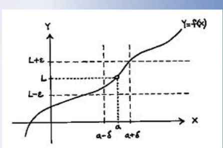
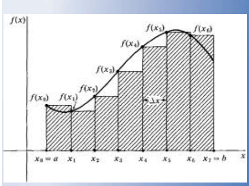



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{\max \Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

# Higher Order Derivatives

$$f'(x), f''(x), f'''(x), f^{iv}(x)$$

$$\frac{dy}{dx}, \quad \frac{d^2y}{dx^2}, \quad \frac{d^3y}{dx^3}, \quad \frac{d^4y}{dx^4}$$

$$y', \quad y'', \quad y''', \quad y^{(4)}$$

$$D_x(y), \quad D_x^2(y), \quad D_x^3(y) \quad D_x^4(y)$$

$$\text{Note that } \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \text{ or } \frac{dy'}{dx}$$

Higher Order Derivatives

| Derivative | $f'$ notation | $y'$ notation | $D_x$ notation | Leibniz notation |
|---|---|---|---|---|
| First | $f'(x)$ | $y'$ | $D_x(f)$ | $\frac{dy}{dx}$ |
| Second | $f''(x)$ | $y''$ | $D_x^2(f)$ | $\frac{d^2y}{dx^2}$ |
| Third | $f'''(x)$ | $y'''$ | $D_x^3(f)$ | $\frac{d^3y}{dx^3}$ |
| Fourth | $f^{(4)}(x)$ | $y^{(4)}$ | $D_x^4(f)$ | $\frac{d^4y}{dx^4}$ |
| Fifth | $f^{(5)}(x)$ | $y^{(5)}$ | $D_x^5(f)$ | $\frac{d^5y}{dx^5}$ |
| $n^{th}$ | $f^{(n)}(x)$ | $y^{(n)}$ | $D_x^n(f)$ | $\frac{d^ny}{dx^n}$ |

$\frac{d^2(y)}{dx^2}$

$y^4 = y$ to the 4th power $\neq y^{(4)}$ is the fourth derivative of $y$

EX 1 Find $f'''(x)$ for $f(x) = (3-5x)^5$

$$f'(x) = 5(3-5x)^4(-5) = -25(3-5x)^4$$

$$f''(x) = -25(4)(3-5x)^3(-5) = 500(3-5x)^3$$

$$f'''(x) = 500(3)(3-5x)^2(-5)$$
$$= -7500(3-5x)^2$$

Ex 2 Find $\frac{dy}{dx}$ for $y = sin\left(\frac{\pi}{x}\right)$ .

$\frac{\pi}{x} = \pi x^{-1}$

$y' = \cos\left(\frac{\pi}{x}\right)\left(-\pi x^{-2}\right)$

Ex 3 What is $D_x^5(3x^4 - 2x^3 + x^2 - 4)$ ?

the first derivative will be a $3^{rd}$ degree poly.
" second " " " $2^{nd}$ " "
" third " " " $1^{st}$ " "
" fourth " " " constant

$\Rightarrow D_x^5(3x^4 - 2x^3 + x^2 - 4) = 0$

Ex 4 Find a formula for $D_x^n\left(\frac{1}{x}\right)$ .

$3! = 3 \cdot 2 \cdot 1$
$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$n! = n(n-1) \cdots 2 \cdot 1 \quad , n \in \mathbb{Z}^+$

$y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$

$y'' = -(-2)x^{-3} = 2x^{-3}$

$y''' = 2(-3)x^{-4} = -6x^{-4} = -3!x^{-4}$

$y^{(4)} = -3!(-4)x^{-5} = 4!x^{-5}$

$y^{(5)} = 4!(-5)x^{-6} = -5!x^{-6}$

| $n$ | $D_x^n\left(\frac{1}{x}\right)$ |
|---|---|
| 1 | $-x^{-2} = -1!x^{-2}$ |
| 2 | $2!x^{-3}$ |
| 3 | $-3!x^{-4}$ |
| 4 | $4!x^{-5}$ |
| 5 | $-5!x^{-6}$ |
| $\vdots$ | |
| $n$ | $(-1)^n n! x^{-(n+1)}$ |
| | $= (-1)^n n! x^{-n-1}$ |

$y^{(20)} = (-1)^{20} 20! x^{-21} = 20! x^{-21}$

We know $v(t) = s'(t)$ $\quad$ velocity = $\dfrac{\text{change in dist.}}{\text{change in time}}$

$a(t) = v'(t) = s''(t)$

$\dfrac{dv}{dt}$

EX 5 An object moves along a horizontal coordinate line according to $s(t)=t^3-6t^2$ .

$s$ is the directed distance from the origin (in ft.) $t$ is the time (in seconds.)

a) What are v(t) and a(t)?

$v(t) = s'(t) = 3t^2 - 12t$ ft/sec

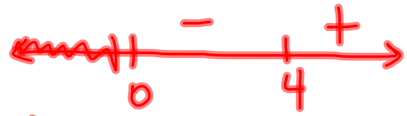
$a(t) = v'(t) = 6t - 12$ ft/sec²

b) When is the object moving to the right? $(v(t) > 0)$

$3t^2 - 12t > 0$

$3t(t-4) > 0$

sign line




$\Rightarrow v(t) > 0$ when $t > 4$ sec

c) When is it moving to the left?

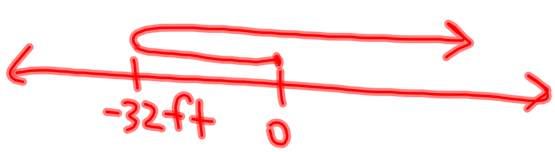
when $v(t) < 0$, when $0 < t < 4$

d) When is its acceleration negative?

$6t - 12 < 0$

$6t < 12$ $\rightarrow$ $t < 2$ sec

e) Draw a schematic diagram that shows the motion of the object.




change directns at $t=4$

$s(4) = 4^3 - 6(4^2)$

$= -32$

$$f'(x), f''(x), f'''(x), f^{iv}(x)$$

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$$D_x(y), \quad D_x^2(y), \quad D_x^3(y) \quad D_x^4(y)$$

$$Note\ that\ \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) or\ \frac{dy'}{dx}$$