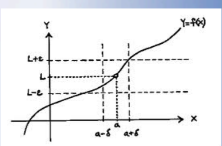
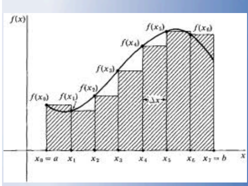
Rules For Finding Derivatives

$$\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2} = \frac{\text{lo}\cdot\text{d hi}-\text{hi}\cdot\text{d lo}}{\text{lo}\cdot\text{lo}}$$



$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$$



$$\lim_{\max \Delta x_i\to 0}\sum_1^n f(x_i)\Delta x_i=\int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx=F(b)-F(a)$$

Constant Function Rule

$y = c$ (c fixed number), then $y' = 0$

($y=c$ graphs into horizontal line, $y'=0$ means slope is zero)

Identity Function Rule

$y = x$, then $y' = 1$

Constant Multiple Rule

$f(x) = k\,g(x)$, k constant, then $f'(x) = k\,g'(x)$

Sum & Difference Rule

$$D_x\big(f(x) \pm g(x)\big) = D_x\big(f(x)\big) \pm D_x\big(g(x)\big)$$

$$\big(f'(x) \pm g'(x)\big)$$

notation for derivative

Notation

$y = f(x)$ Derivative

y'

$f'(x)$

$D_x(y)$

$\frac{dy}{dx}$

POWER RULE

n	$f(x)=x^n$	$f'(x)$
0	$f(x)=1$	$f'(x)=0$
1	$f(x)=x$	$f'(x)=1$
2	$f(x)=x^2$	$f'(x)=2x$
3	$f(x)=x^3$	$f'(x)=3x^2$
4	$f(x)=x^4$	$f'(x)=4x^3$
⋮		
n	$f(x)=x^n$	$\boxed{f'(x)=nx^{n-1}}$

$f(x)=x^2$

② $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}2x+h$$

$$=2x$$

③ $n=3$, $f(x)=x^3$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(3x^2+3x\cancel{h}+\cancel{h^2})$$

$$=3x^2$$

Power Rule

n is any positive integer, $f(x)=x^n$

$$f'(x)=nx^{n-1}$$

EX 1 Find $f'(x)$ if $f(x) = 3x^7-4x^6+x^5+2x^3-x^2+4$

$$f'(x) = D_x(3x^7-4x^6+x^5+2x^3-x^2+4)$$

$$= D_x(3x^7) - D_x(4x^6) + D_x(x^5) + D_x(2x^3) - D_x(x^2) + D_x(4)$$

$$= 3D_x(x^7) - 4D_x(x^6) + D_x(x^5) + 2D_x(x^3) - D_x(x^2) + 0$$

$$= 3(7x^6) - 4(6x^5) + 5x^4 + 2(3x^2) - 2x$$

$$= 21x^6 - 24x^5 + 5x^4 + 6x^2 - 2x$$

<u>Product Rule</u>

If f and g are differentiable, then

$$D_x\big(f(x)g(x)\big) = f(x)D_x[g(x)] + D_x[f(x)]g(x)$$

derivative of a product is the sum of the derivative of one fn times the other fn plus the deriv. of other fn times the first fn

Note: must use when differentiating the product of fns.

EX 2 Find *f'(x)* if $f(x) = (\underbrace{2x^3-4x+1}_{\textcircled{1}})(\underbrace{3x+5}_{\textcircled{2}})$.

a) Use the product rule:

$$f'(x) = \overset{D\textcircled{1}}{(6x^2-4)}\overset{\textcircled{2}}{(3x+5)} + \overset{\textcircled{1}}{(2x^3-4x+1)}\overset{D\textcircled{2}}{(3)}$$

$$= 18x^3+30x^2-12x-20+6x^3-12x+3$$

$$= 24x^3+30x^2-24x-17$$

b) Multiply out and use the power rule to check:

$$f(x) = 6x^4+10x^3-12x^2-20x+3x+5$$

$$f(x) = 6x^4+10x^3-12x^2-17x+5$$

$$\Rightarrow f'(x) = 24x^3+30x^2-24x-17$$

Quotient Rule

Let f and g be differentiable functions, g(x) ≠ 0,

then $D_x \dfrac{f(x)}{g(x)} = \dfrac{g(x)D_x[f(x)] - f(x)D_x[g(x)]}{g^2(x)}$.

$$\frac{\text{lo d-hi} - \text{hi d-lo}}{\text{lo}^2}$$

EX 3 Find f'(x) if $f(x) = \dfrac{2x^2 + 4x - 1}{3x - 2}$.

$$f'(x) = \frac{(3x-2)(4x+4) - (2x^2+4x-1)(3)}{(3x-2)^2}$$

Note:

EX 4 $y = \frac{-3}{x} + \frac{2}{x^4 - 7x}$ Find $y'(x)$

$y = -3x^{-1} + \frac{2}{x^4 - 7x}$

$y' = -3(-1)x^{-2} + \frac{(x^4-7x)(0) - 2(4x^3-7)}{(x^4-7x)^2}$

$= 3x^{-2} + \frac{-2(4x^3-7)}{(x^4-7x)^2}$

$D_x\left(x^{-n}\right) = -nx^{-n-1}$ for n, a positive integer

$D_x(x^{-n}) = D_x\left(\frac{1}{x^n}\right)$

$= \frac{x^n(0) - 1(nx^{n-1})}{(x^n)^2}$

$= \frac{-nx^{n-1}}{x^{2n}} = -nx^{n-1-2n}$

$= -nx^{-n-1}$

EX 5 Find $f'(x)$ if $f(x)=\dfrac{5x-4}{x^2+1}$ (quotient rule)

$$f'(x)=\frac{(x^2+1)(5)-(5x-4)(2x)}{(x^2+1)^2}$$

EX 6 Find $D_x(y)$ if $y=3x(x^3-2x+1)$ $=3x^4-6x^2+3x$

$$D_x(y)=y'=12x^3-12x+3$$

EX 7 Find $\dfrac{dy}{dx}$ if $y=\dfrac{-3}{x^5}+\dfrac{2}{x}$ $=-3x^{-5}+2x^{-1}$

$$\frac{dy}{dx}=-3(-5)x^{-6}+2(-1)x^{-2}$$

$$=15x^{-6}-2x^{-2} \text{ or } \frac{15}{x^6}-\frac{2}{x^2}$$

$$\frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$
$$=\frac{\mathrm{lo}\cdot\mathrm{d\ hi}-\mathrm{hi}\cdot\mathrm{d\ lo}}{\mathrm{lo}\cdot\mathrm{lo}}$$